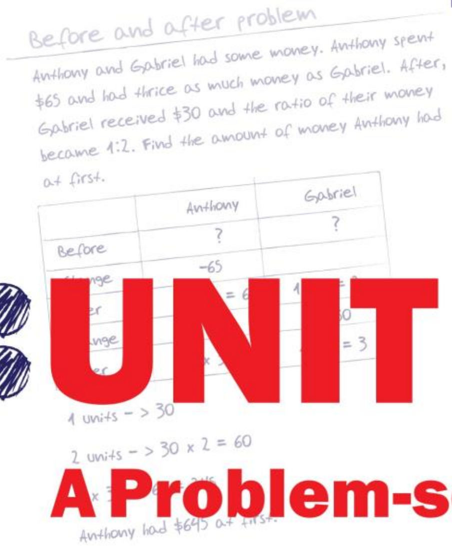
Mastering Heuristics Series
Handbooks for discerning parents

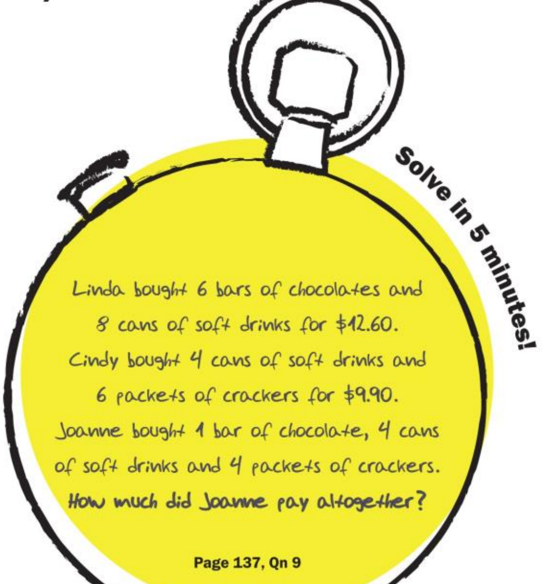


2nd edition

1:UNIT TRANSFER METHOD

A Problem-solving Tool

for Challenging Problems in Upper Primary Mathematics (Primary 5 & 6)



Sunny Tan

Mastering Heuristics Series
Handbook for discerning parents

Unit Transfer Method (2nd Edition)
A Problem-solving Tool
for Challenging Problems
in Upper Primary Mathematics
(Primary 5 & 6)

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Maths Heuristics Private Limited

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PREFACE

Heuristics in Primary Maths Syllabus

Heuristics is a specialised mathematical problem-solving concept. Mastering it facilitates efficiency in solving regular as well as challenging mathematical problems. The Ministry of Education in Singapore has incorporated 11 Problem-Solving Heuristics into all primary-level mathematical syllabus.

Learning Heuristics Effectively

Instead of containing the 11 Problem-Solving Heuristics neatly into specific chapters though, they have been integrated into the regular curriculum. This not only makes it difficult for students to pick up Heuristics skills, but can also make mathematics confusing for some students. For us parents, it is difficult for us to put aside the regular-syllabus mathematical concepts we were brought up on to re-learn Heuristics, much less teach our own children this new concept.

Take Algebraic Equations, for instance. Although it is a Heuristics technique, the topic has never been, and is still not, taught at primary level. Yet, primary-level Mathematics Papers these days include questions from the topic. Parents, being familiar with the topic, will attempt to teach their children to solve the question using Algebraic Equations. This will only confuse their children. According to current primary-level mathematical syllabus, other Heuristics techniques should be used instead.

These and other challenges were what I observed first hand during my years as a mathematics teacher, and what provided me the impetus for my post-graduate studies, mathsHeuristics™ programmes and, now, the Mastering Heuristics Series of books.

About Mastering Heuristics Series

This series of books is a culmination of my systematic thinking, supported by professional instructional writing and editing, to facilitate understanding and mastery of Heuristics. Through it, I have neatly packaged Heuristics into logical topics (Series of books) and sub-topics (Chapters within each book). For each sub-topic, I offer many examples, showing how the sub-topic may be applied, and then explaining the application in easy-to-follow steps and visualisations without skipping a beat.

This particular book in the series deals specifically with Unit Transfer Method, the use of ratio to effectively analyse and solve challenging mathematical problems involving whole numbers, fractions, decimals, percentages and ratios. This simple, logical yet powerful problem-solving technique complements the model approach and the algebraic approach.

The entire series of four books provides a complete and comprehensive guide to Heuristics. While each book introduces parents to a few Heuristics topics, it gives students the opportunity to see how the specific Heuristics work as well as get in some practice.

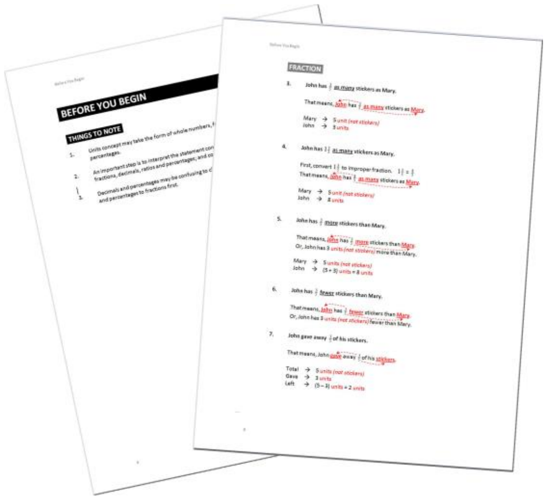
For students enrolled in mathsHeuristics™ programmes, each book serves as a great companion, while keeping parents well-informed of what their children are learning.

Sunny Tan

November 2010

HOW TO USE THIS BOOK

BEFORE YOU BEGIN

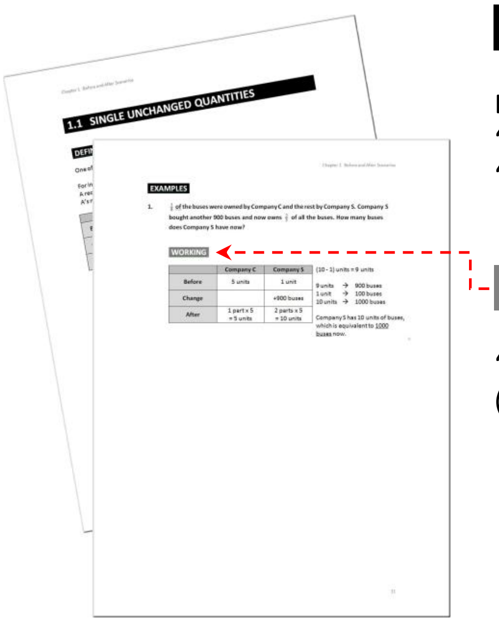


The “Before You Begin” chapter instills the basic but important step that must be applied across every question under this topic. This helps to standardise the given information for easy application of the techniques being taught.

In this book on Unit Transfer Method (UTM), this step is to convert whole numbers, fractions, decimals, percentages and ratios into units.

CHAPTERS AND SECTIONS

The various UTM techniques are neatly separated into different chapters and sections. Thus, examples of UTM application are classified according to problem-solving techniques for more focused learning.

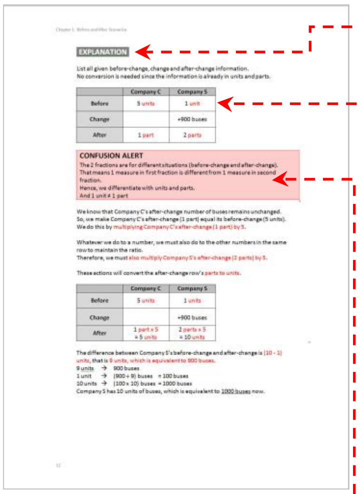


EXAMPLES

Each example of UTM application comes with “Working” and “Explanation”, which includes “Confusion Alert” boxes.

WORKING

“Working” shows heuristics application in action (how quick it is to solve a question).



EXPLANATION

"Explanation" shows the thought process behind the heuristics application (the detailed steps). It takes readers through the solution in the following manner:

- step-by-step without skipping a beat so that readers can follow what happens at each and every step.
- systematic so that readers begin to see a pattern in applying the technique.
- easy-to-follow so that readers can quickly understand the technique minus the frustration.

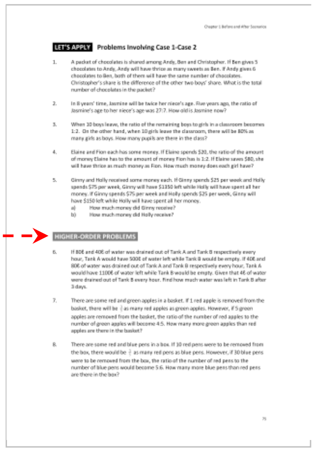
In UTM, readers will see that its application always begins with:

- the basic step explained in "Before You Begin", and
- the tabulation of all given information.

This quickly helps students see and understand the relationships among all the information given in the question.

"Confusion Alert" boxes in the midst of "Explanation" highlight areas where students are likely to be uncertain of or make mistakes in. It also gives the rationale to help clarify doubts in these areas.

LET'S APPLY



Learning is only effective when followed up with practice. Hence, at the end of each chapter/section is a list of questions related to the heuristics technique taught in that chapter/section.

HIGHER-ORDER PROBLEMS

Higher-order questions have also been added to every practice section. This aims to stretch students' skills in applying the heuristics technique.

ADDITIONAL TIPS

For on-going sharing and discussions on the use of UTM, visit:
www.unittransfermethod.blogspot.com

For detailed workings to all UTM "Let's Apply" and "Higher-order Problems" sections, visit:
www.mathsHeuristics.com/solution-s1-utm.html

BEFORE YOU BEGIN

THINGS TO NOTE

1. Units concept may take the form of whole numbers, fractions, decimals, ratios or percentages.

2. An important step is to interpret the statement containing whole numbers, fractions, decimals, ratios and percentages; and convert them into units.

3. Decimals and percentages may be confusing to children. So, convert any decimals and percentages to fractions first.

EXAMPLES

WHOLE NUMBER

1. **John has 5 times <u>as many</u> stickers as Mary.**

First, convert the whole number to fraction. $5 = \frac{5}{1}$

That means, <u>John</u> has $\frac{5}{1}$ times <u>as many</u> stickers as <u>Mary</u>.

Mary → 1 unit *(not stickers)*

John → 5 units

2. **John has 5 times <u>more</u> stickers than Mary.**

First, convert the whole number to fraction. $5 = \frac{5}{1}$

That means, <u>John</u> has $\frac{5}{1}$ times <u>more</u> stickers than <u>Mary</u>.

Mary → 1 unit *(not stickers)*

John → (1 + 5) units = 6 units

FRACTION

3. **John has $\frac{3}{5}$ <u>as many</u> stickers as Mary.**

That means, **<u>John</u>** has $\frac{3}{5}$ **<u>as many</u>** stickers as **<u>Mary</u>**.

Mary → 5 units *(not stickers)*
John → 3 units

4. **John has $1\frac{3}{5}$ <u>as many</u> stickers as Mary.**

First, convert $1\frac{3}{5}$ to improper fraction. $1\frac{3}{5} = \frac{8}{5}$
That means, **<u>John</u>** has $\frac{8}{5}$ **<u>as many</u>** stickers as **<u>Mary</u>**.

Mary → 5 units *(not stickers)*
John → 8 units

5. **John has $\frac{3}{5}$ <u>more</u> stickers than Mary.**

That means, **<u>John</u>** has $\frac{3}{5}$ **<u>more</u>** stickers than **<u>Mary</u>**.
Or, John has 3 units *(not stickers)* more than Mary.

Mary → 5 units *(not stickers)*
John → (5 + 3) units = 8 units

6. **John has $\frac{3}{5}$ <u>fewer</u> stickers than Mary.**

That means, **<u>John</u>** has $\frac{3}{5}$ **<u>fewer</u>** stickers than **<u>Mary</u>**.
Or, John has 3 units *(not stickers)* fewer than Mary.

Mary → 5 units *(not stickers)*
John → (5 – 3) units = 2 units

7. **John gave away $\frac{3}{5}$ of his stickers.**

That means, John **<u>gave</u>** away $\frac{3}{5}$ of his **<u>stickers</u>**.

Total → 5 units *(not stickers)*
Gave → 3 units
Left → (5 – 3) units = 2 units

DECIMAL

8. John has 0.6 times <u>as many</u> stickers as Mary.

First, convert the decimal to fraction. 0.6 = $\frac{6}{10}$ = $\frac{3}{5}$

That means, **John** has $\frac{3}{5}$ **as many** stickers as **Mary**. **(Just like Example 3)**

Mary → 5 units *(not stickers)*
John → 3 units

9. John has 1.6 times <u>as many</u> stickers as Mary.

First, convert the decimal to improper fraction. 1.6 = $1\frac{6}{10}$ = $1\frac{3}{5}$ = $\frac{8}{5}$

That means, **John** has $\frac{8}{5}$ **as many** stickers as **Mary**. **(Just like Example 4)**

Mary → 5 units *(not stickers)*
John → 8 units

10. John has 0.6 times <u>more</u> stickers than Mary.

First, convert the decimal to fraction. 0.6 = $\frac{6}{10}$ = $\frac{3}{5}$

That means, **John** has $\frac{3}{5}$ **more** stickers than **Mary**. **(Just like Example 5)**

Or, John has 3 units *(not stickers)* more than Mary.

Mary → 5 units *(not stickers)*
John → (5 + 3) units = 8 units

11. John has 0.6 times <u>fewer</u> stickers than Mary.

First, convert the decimal to fraction. 0.6 = $\frac{6}{10}$ = $\frac{3}{5}$

That means, **John** has $\frac{3}{5}$ **fewer** stickers than **Mary**. **(Just like Example 6)**

Or, John has 3 units *(not stickers)* fewer than Mary.

Mary → 5 units *(not stickers)*
John → (5 – 3) units = 2 units

12. John gave away 0.6 of his stickers.

First, convert the decimal to fraction. 0.6 = $\frac{6}{10}$ = $\frac{3}{5}$

That means, John **gave** away $\frac{3}{5}$ of his **stickers**. **(Just like Example 7)**

Total → 5 units *(not stickers)*
Gave → 3 units
Left → (5 – 3) units = 2 units

PERCENTAGE

13. **John has 60% <u>as many</u> stickers as Mary.**

First, convert the percentage to fraction. 60% = $\frac{60}{100}$ = $\frac{3}{5}$

That means, **<u>John</u>** has $\frac{3}{5}$ **<u>as many</u>** stickers as **<u>Mary</u>**. **(Just like Example 3)**

Mary → 5 units *(not stickers)*

John → 3 units

14. **John has 160% <u>as many</u> stickers as Mary.**

First, convert the percentage to improper fraction. 160% = $\frac{160}{100}$ = $\frac{8}{5}$

That means, **<u>John</u>** has $\frac{8}{5}$ **<u>as many</u>** stickers as **<u>Mary</u>**. **(Just like Example 4)**

Mary → 5 units *(not stickers)*

John → 8 units

15. **John has 60% <u>more</u> stickers than Mary.**

First, convert the percentage to fraction. 60% = $\frac{60}{100}$ = $\frac{3}{5}$

That means, **<u>John</u>** has $\frac{3}{5}$ **<u>more</u>** stickers than **<u>Mary</u>**. **(Just like Example 5)**

Or, John has 3 units *(not stickers)* more than Mary.

Mary → 5 units *(not stickers)*

John → (5 + 3) units = 8 units

16. **John has 60% <u>fewer</u> stickers than Mary.**

First, convert the percentage to fraction. 60% = $\frac{60}{100}$ = $\frac{3}{5}$

That means, **<u>John</u>** has $\frac{3}{5}$ **<u>fewer</u>** stickers than **<u>Mary</u>**. **(Just like Example 6)**

Or, John has 3 units *(not stickers)* fewer than Mary.

Mary → 5 units *(not stickers)*

John → (5 – 3) units = 2 units

17. **John <u>gave</u> away 60% of his stickers.**

First, convert the percentage to fraction. 60% = $\frac{60}{100}$ = $\frac{3}{5}$

That means, John **<u>gave</u>** away $\frac{3}{5}$ of his **<u>stickers</u>**. **(Just like Example 7)**

Total → 5 units *(not stickers)*

Gave → 3 units

Left → (5 – 3) units = 2 units

RATIO

18. John and Mary have stickers in the ratio of 3:5.

John and **Mary** have stickers in the **ratio** of 3:5.

Mary → 5 units *(not stickers)*
John → 3 units

CHAPTER 1 BEFORE AND AFTER SCENARIOS

STEPS

- List all given before-action, change-action and after-action information.
- Convert the information into units and parts,
  where necessary and if not already in units and parts.
- Compare the information to find the unknown.

APPLICABILITY

There are five basic scenarios where the Before and After Concept may be applied.

- Single Unchanged Quantities
- Total Unchanged Quantities
- Difference Unchanged Quantities
- All Changing Quantities
- Case 1-Case 2

In the All Changing Quantities scenario, a modified version of Unit Transfer Method is used to solve the problem.

Each Case 1-Case 2 scenario is effectively one of the preceding four Before and After scenarios, except that the Case 1-Case 2 scenario requires higher-order thinking skills to be solved.

1.1 SINGLE UNCHANGED QUANTITIES

DEFINITION

One of the given quantities remains unchanged.

For instance, A and B have stickers in a certain quantities (Say, 20:50).
A receives 5 stickers from C (external party).
A's number of stickers changed, B's number of stickers remains unchanged.

	A	B
Before	20	50
Change	+5	
After	25	50

→ In the change row, the change figure appears in the column where the change occurred.

↓

In the column where the change cell is empty (no change occurred), the before-change and after-change quantities remain the same (quantities unchanged).

While A's number of stickers changes (+5), B's number of stickers remains unchanged (Single Unchanged Quantity).

EXAMPLES

1. $\frac{5}{6}$ **of the buses were owned by Company C and the rest by Company S. Company S bought another 900 buses and now owns** $\frac{2}{3}$ **of all the buses. How many buses does Company S have now?**

WORKING

	Company C	**Company S**
Before	5 units	1 unit
Change		+900 buses
After	1 part x 5 = 5 units	2 parts x 5 = 10 units

(10 - 1) units = 9 units

9 units → 900 buses
1 unit → (900 ÷ 9) buses = 100 buses
10 units → (100 x 10) buses = 1000 buses

Company S has 10 units of buses, which is equivalent to <u>1000 buses</u> now.

EXPLANATION

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Company C	Company S
Before	5 units	1 unit
Change		+900 buses
After	1 part	2 parts

CONFUSION ALERT
The 2 fractions are for different situations (before-change and after-change).
That means 1 measure in first fraction is different from 1 measure in second fraction.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that Company C's after-change number of buses remains unchanged.
So, we make Company C's after-change (1 part) equal its before-change (5 units).
We do this by multiplying Company C's after-change (1 part) by 5.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must also multiply Company S's after-change (2 parts) by 5.

These actions will convert the after-change row's parts to units.

	Company C	Company S
Before	5 units	1 units
Change		+900 buses
After	1 part x 5 = 5 units	2 parts x 5 = 10 units

The difference between Company S's before-change and after-change is (10 - 1) units, that is 9 units, which is equivalent to 900 buses.
9 units → 900 buses
1 unit → (900 ÷ 9) buses = 100 buses
10 units → (100 x 10) buses = 1000 buses
Company S has 10 units of buses, which is equivalent to <u>1000 buses</u> now.

2. **At a gathering, the ratio of the number of men to the number of women was 5:7. After 60 men left the gathering, the new ratio of the number of men to the number of women became 1:2. How many men were there at first?**

WORKING

	Men	Women
Before	5 units x 2 = 10 units	7 units x 2 = 14 units
Change	- 60 men	
After	1 part x 7 = 7 units	2 parts x 7 = 14 units

(10 - 7) units = 3 units

3 units → 60 men
1 unit → (60 ÷ 3) men = 20 men
10 units → (20 x 10) men = 200 men

There were 10 units of men, which is equivalent to <u>200 men</u> at first.

EXPLANATION

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Men	Women
Before	5 units	7 units
Change	- 60 men	
After	1 part	2 parts

CONFUSION ALERT

The 2 ratios are for different situations (before-change and after-change).
That means 1 measure in first ratio is different from 1 measure in second ratio.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that the number of women remains unchanged.
So, we make the women's after-change (2 parts) equal their before-change (7 units).
We do this by multiplying the women's after-change (2 parts) by 7,
and their before-change (7 units) by 2.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must also multiply the men's after-change (1 part) by 7,
and their before-change (5 units) by 2.

These actions will convert the after-change row's parts to units.

	Men	Women
Before	5 units x 2 = 10 units	7 units x 2 = 14 units
Change	- 60 men	
After	1 part x 7 = 7 units	2 parts x 7 = 14 units

The difference between the men's before-change and after-change is (10 - 7) units, that is 3 units, which is equivalent to 60 men.
3 units → 60 men
1 unit → (60 ÷ 3) men = 20 men
10 units → (20 x 10) men = 200 men
There were 10 units of men, which is equivalent to 200 men at first.

3. **Kane has 90 stamps in his album. 60% of them are from Singapore while the rest are from Thailand. After giving away some Singapore stamps, the percentage of Singapore stamps reduces to 55%. How many Singapore stamps does he have in the end?**

WORKING

60% = $\frac{60}{100} = \frac{3}{5}$ 55% = $\frac{55}{100} = \frac{11}{20}$

	Singapore Stamps	**Thai Stamps**	**Total stamps**
Before	3 units ↓ 3 units x 9 = 27 units	(5 - 3) units = 2 units ↓ 2 units x 9 = 18 units	5 units ↓ 5 units x 9 = 45 units → 90 stamps
Change	- ? stamps		
After	11 parts ↓ 11 parts x 2 = 22 units	(20 - 11) parts = 9 parts ↓ 9 parts x 2 = 18 units	

45 units → 90 stamps
1 unit → (90 ÷ 45) stamps = 2 stamps
22 units → (2 x 22) stamps = 44 stamps

Kane has 22 units of Singapore stamps, which is equivalent to 44 Singapore stamps in the end

EXPLANATION

Smaller numbers are more manageable.
So, convert the percentage to fraction first.

60% = $\frac{60}{100} = \frac{3}{5}$ 55% = $\frac{55}{100} = \frac{11}{20}$

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Singapore stamps	Thai stamps	Total stamps
Before	3 units	(5 - 3) units = 2 units	5 units
Change	- ? stamps		
After	11 parts	(20 - 11) parts = 9 parts	

CONFUSION ALERT

The 2 fractions are for different situations (before-change and after-change).
That means 1 measure in first fraction is different from 1 measure in second fraction.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that the number of Thai stamps remains unchanged.
So, we make the Thai stamps' after-change (9 units) equal its before-change (2 units).
We do this by multiplying the Thai stamps' after-change (9 parts) by 2,
and its before-change (2 units) by 9.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must multiply the Singapore stamps' after-change (11 parts) by 2,
its before-change (3 units) by 9,
and the total stamps before-change (5 units) by 9.

These actions will convert the after-change row's parts to units.

	Singapore Stamps	Thai Stamps	Total stamps
Before	3 units x 9 = 27 units	2 units x 9 = 18 units	5 units x 9 = 45 units → 90 stamps
Change	- ? stamps		
After	11 parts x 2 = 22 units	9 parts x 2 = 18 units	

45 units → 90 stamps
1 unit → (90 ÷ 45) stamps = 2 stamps
22 units → (2 x 22) stamps = 44 stamps
Kane has 22 units of Singapore stamps, which is equivalent to <u>44 Singapore stamps</u> in the end.

4. **In a fruit stall, the ratio of number of apples to number of oranges is 2:9. The ratio of number of oranges to number of pears is 3:4. The next day, 135 oranges were sold, $\frac{4}{11}$ of the remaining fruits in the stall were oranges. How many more pears than apples were there in the stall at first?**

WORKING

	Apples	Oranges	Pears
Before	2 units	9 units	
		3 parts x 3 = 9 units	4 parts x 3 = 12 units

	Oranges	Apples & Pears
Before	9 units	(2 + 12) units = 14 units
Change	- 135 oranges	
After	4 parts x 2 = 8 units	7 parts x 2 = 14 units

(9 - 8) units = 1 unit → 135 fruits

There were (12 – 2) units, that is 10 units more pears than apples at first.

1 unit → 135 fruits
10 units → (135 x 10) fruits = 1350 fruits

There were 10 units, which is equivalent to 1350 more pears than apples at first.

EXPLANATION

List only the before-change information for now.
No conversion is needed since the information is already in units and parts.

	Apples	Oranges	Pears
Before	2 units	9 units	
		3 parts	4 parts

CONFUSION ALERT
The 2 ratios are for different situations
(apples compared to oranges, and oranges compared to pears).
That means 1 measure in first ratio is different from 1 measure in second ratio.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that the oranges in both ratios are the same objects.
So, we make the oranges (3 parts) in the oranges to pears ratio equal 9 units.
We do this by multiplying the oranges (3 parts) in the oranges to pears ratio by 3.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must multiply the pears (4 parts) in the oranges to pears ratio by 3.

These actions will convert the oranges to pears ratio's parts to units.

	Apples	Oranges	Pears
Before	2 units	9 units	
		3 parts x 3 = 9 units	4 parts x 3 = 12 units

Next, we merge the tables.
Then list all other information, that is all the change and after-change information.
No conversion is needed since the information is already in units and parts.

	Oranges	Apples & Pears
Before	9 units	(2 + 12) units = 14 units
Change	- 135 oranges	
After	4 parts	7 parts

CONFUSION ALERT

The 2 ratios are for different situations (before-change and after-change).
That means 1 measure in first ratio is different from 1 measure in second ratio.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

Also note that earlier use of parts in the orange to pear ratio is different from current use of parts in the after-change ratio.

We know that the total number of apples and pears remains unchanged.
So, we make the apples and pears' after-change (7 parts) equal their before-change (14 units).
We do this by multiplying the apples and pears' after-change (7 parts) by 2.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must multiply the oranges' after-change (4 parts) by 2.

These actions will convert the after-change row's parts to units.

	Oranges	**Apples & Pears**
Before	9 units	14 units
Change	- 135 oranges	
After	4 parts x 2 = 8 units	7 parts x 2 = 14 units

The difference between the oranges' before-change and after-change is (9 - 8) units, that is 1 unit, which is equivalent to 135 fruits.

At first, there were 2 units of apples and 12 units of pears.
That means, there were (12 - 2) units, that is 10 units more pears than apples at first.

1 unit → 135 fruits
10 units → (135 x 10) fruits = 1350 fruits

There were 10 units, which is equivalent to <u>1350</u> more pears than apples at first.

LET'S APPLY Problems Involving Single Unchanged Quantities

1. $\frac{1}{5}$ of the children at the playground were girls and the rest were boys. When 8 girls left the playground, the fraction of girls decreased to $\frac{1}{7}$ of the total number of children at the playground. How many children were at the playground at first?

2. There is a box full of pebbles and seashells. Pat put in an additional 80 pebbles into the box and the percentage of pebbles increased from 10% to 30%. How many more seashells than pebbles were there in the box in the end?

3. There was a total of 440 sparrows and pigeons in a park. 25% of these birds were pigeons. When some sparrows left the park, the percentage of pigeons in the park increased to 55%. How many sparrows were left in the park?

4. Mrs Liew baked 3 times as many apple pies as cakes. If she had baked 60 fewer apple pies, she would have baked twice as many cakes as apple pies.
   a) How many apple pies did she bake?
   b) How many cakes did she bake?

5. A concert hall has 600 seats. 10% of the seats are VIP seats while the rest are regular seats. How many VIP seats must be added so that the number of VIP seats is increased to 20%?

HIGHER-ORDER PROBLEMS

6. Three sisters, Angie, Bernice and Candice share some sweets. Angie's share is 40% of the total number of sweets the three sisters have. Bernice has 40 sweets more than Angie. Bernice's share is 4 times Candice's. Then Angie and Bernice gives an equal amount of sweets to their younger brother and the new ratio between the three sisters becomes 5:7:3. How many sweets were given to their younger brother?

7. Three children went on a trick-or-treat trip to collect candies. At the end of the day, the ratio of Caline's candies to Denise's candies is 2:3 and the ratio of Ella's candies to Denise's candies is 6:7. Later on, Ella and Caline donated half of their respective shares to the orphanage in a bag with 64 candies in it. Find out the total number of candies the three children have after the donation.

8. Tom had an album of Singapore and Malaysian stamps. When he gave away 50 Singapore stamps, there were 2.5 times as many Singapore stamps as Malaysian stamps. After his brother gave him another 63 Malaysian stamps, he had 2.5 times as many Malaysian stamps as Singapore stamps. How many more Singapore stamps than Malaysian stamps were there in the Tom's album at first?

9. Alice spent $\frac{3}{5}$ of her monthly allowance. The next month, her monthly allowance increased by $15 and she saved 25% of her new monthly allowance. If what she saved in both months were the same amount, find her new monthly allowance.

1.2 TOTAL UNCHANGED QUANTITIES

DEFINITION

The total quantity remains unchanged.

For instance, A and B have stickers in a certain quantities (Say, 20:30).
A gives B 5 stickers (internal party).
A and B's number of stickers changes,
but A and B's total number of stickers remain unchanged before-transfer and after-transfer.

	A	B	Total
Before	20	30	50
Change	-5	+5	
After	15	35	50

→ In the A and B change row, the change figure appears in both A and B columns.
While the quantities are the same,
the signs are different (+/-).

↓
A simply made a transfer to B.
This does not change A and B's total.
So, the change cell is empty (no change occurred).
And the before-change and after-change Total
remain the same (Total Unchanged Quantities).

EXAMPLES

1. **Margaret has 20% more stickers than Ben. After receiving some stickers from Ben, the ratio of Margaret's stickers to Ben's stickers becomes 3:1. Find the number of stickers Margaret received from Ben if she has 132 stickers in the end.**

WORKING

	Margaret's stickers	Ben's stickers	Total stickers
Before	(5 + 1) units = 6 units ↓ 6 units x 4 = 24 units	5 units ↓ 5 units x 4 = 20 units	(6 + 5) units = 11 units ↓ 11 units x 4 = 44 units
Change	Received from Ben (33 - 24) units = 9 units	Gave to Margaret (20 - 11) units = 9 units	
After	3 parts ↓ 3 parts x 11 = 33 units → 132 stickers	1 part ↓ 1 part x 11 = 11 units	(3 + 1) parts = 4 parts ↓ 4 parts x 11 = 44 units

33 units → 132 stickers
1 unit → (132 ÷ 33) stickers = 4 stickers
9 units → (4 x 9) stickers = 36 stickers

Margaret received 9 units, which is equivalent to <u>36 stickers</u>, from Ben.

EXPLANATION

Smaller numbers are more manageable.
So, convert the percentage to fraction first.

20% = $\frac{20}{100} = \frac{1}{5}$

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Margaret's stickers	Ben's stickers	Total stickers
Before	(5 + 1) units = 6 units	5 units	(6 + 5) units = 11 units
Change	+ ?	- ?	
After	3 parts	1 parts	(3 + 1) parts = 4 parts

CONFUSION ALERT

The percentage and ratio are for different situations (before-change and after-change).
That means 1 measure in before-change situation is different from 1 measure in after-change situation.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that the total number of stickers remains unchanged.
So, we make the total stickers' after-change (4 parts) equal its before-change (11 units).
We do this by multiplying the total stickers' after-change (4 parts) by 11,
and its before-change (11 units) by 4.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must multiply the after-change row (3 parts, 1 part and 4 parts) by 11,
and the before-change row (6 units, 5 units and 11 units) by 4.

These actions will convert the after-change row's parts to units.

	Margaret's stickers	Ben's stickers	Total stickers
Before	6 units x 4 = 24 units	5 units x 4 = 20 units	11 units x 4 = 44 units
Change	+ ?	- ?	
After	3 parts x 11 = 33 units	1 parts x 11 = 11 units	4 parts x 11 = 44 units

Next, we work out the change.

	Margaret's stickers	Ben's stickers	Total stickers
Before	24 units	20 units	44 units
Change	Received from Ben (33 - 24) units = 9 units	Gave to Margaret (20 - 11) units = 9 units	
After	33 units → 132 stickers	11 units	44 units

Margaret has 33 units of stickers, which is equivalent to 132 stickers, after-change.
33 units → 132 stickers
1 unit → (132 ÷ 33) stickers = 4 stickers
9 units → (4 x 9) stickers = 36 stickers
Margaret received 9 units, which is equivalent to 36 stickers, from Ben.

2. **The ratio of the volume of syrup in Jug A to the volume of syrup in Jug B was 3:2. When 64 ml of syrup from Jug A was poured into Jug B, the ratio of the volume of syrup in Jug A to the volume of syrup in Jug B became 1:2. What was the volume of syrup in Jug A in the end?**

WORKING

	Syrup in Jug A	**Syrup in Jug B**	**Total**
Before	3 units ↓ 3 units x 3 = 9 units	2 units ↓ 2 units x 3 = 6 units	(3 + 2) units = 5 units ↓ 5 units x 3 = 15 units
Change	- 64 ml → (9 - 5) units = 4 units	+ 64 ml → (10 - 6) units = 4 units	
After	1 part ↓ 1 part x 5 = 5 units	2 parts ↓ 2 parts x 5 = 10 units	(1 + 2) parts = 3 parts ↓ 3 parts x 5 = 15 units

4 units → 64 ml
1 unit → (64 ÷ 4) ml = 16 ml
5 units → (16 x 5) ml = 80 ml

Jug A has 5 units, which is equivalent to <u>80 ml</u> in the end.

EXPLANATION

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Syrup in Jug A	Syrup in Jug B	Total
Before	3 units	2 units	(3 + 2) units = 5 units
Change	- 64 ml	+ 64 ml	
After	1 part	2 parts	(1 + 2) parts = 3 parts

CONFUSION ALERT

The 2 ratios are for different situations (before-change and after-change).
That means 1 measure in the first ratio is different from 1 measure in the second ratio.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that the total volume remains unchanged.
So, we make the total volume after-change (3 parts) equal its before-change (5 units).
We do this by multiplying the total volume after-change (3 parts) by 5,
and its before-change (5 units) by 3.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must multiply the after-change row (1 part and 2 parts) by 5,
and the before-change row (3 units and 2 units) by 3.

These actions will convert the after-change row's parts to units.

	Syrup in Jug A	Syrup in Jug B	Total
Before	3 units x 3 = 9 units	2 units x 3 = 6 units	5 units x 3 = 15 units
Change	- 64 ml	+ 64 ml	
After	1 part x 5 = 5 units	2 parts x 5 = 10 units	3 parts x 5 = 15 units

The change in volume of syrup in each jug is (9 - 5) **OR** (10 - 6) units, that is 4 units, which is equivalent to 64 ml.

4 units → 64 ml
1 unit → (64 ÷ 4) ml = 16 ml
5 units → (16 x 5) ml = 80 ml

Jug A has 5 units, which is equivalent to 80 ml in the end.

3. **Zen had two boxes of pencils. After transferring $\frac{7}{17}$ of the pencils in Box B to Box A, the ratio of the number of pencils in Box A to that in Box B becomes 3:2. What was the ratio of the number of pencils in Box A to that in Box B at first?**

WORKING

	Pencils in Box A	**Pencils in Box B**	**Total**
Before	? units = (25 - 17) units = 8 units	17 units	25 units
Change	+ 7 units	- 7 units	
After	3 parts ↓ 3 parts x 5 = 15 units	2 parts ↓ 2 parts x 5 = 10 units	(3 + 2) parts = 5 parts ↓ 5 parts x 5 = 25 units

The ratio of the number of pencils in Box A to that in Box B is <u>8:17</u> at first.

EXPLANATION

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Pencils in Box A	Pencils in Box B	Total
Before	? units	17 units	
Change	+ 7 units	- 7 units	
After	3 parts	2 parts → (17 - 7) units = 10 units	(3 + 2) parts = 5 parts

CONFUSION ALERT
The fraction and ratio are for different situations (before-change and after-change).
That means 1 measure in the fraction is different from 1 measure in the ratio.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that the number of pencils in Box B after-change is (17 - 7) units, that is 10 units.
So, we make the number of pencils in Box B after-change (2 parts) equal 10 units.
We do this by multiplying the number of pencils in Box B after-change (2 parts) by 5.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must multiply the after-change row (3 parts and 5 parts) by 5.

These actions will convert the after-change row's parts to units.

	Pencils in Box A	Pencils in Box B	Total
Before	? units	17 units	25 units
Change	+ 7 units	- 7 units	
After	3 parts x 5 = 15 units	2 parts x 5 = 10 units	5 parts x 5 = 25 units

Since the total number of pencils did not change,
the total number of pencils before-change must also be 25 units.
Therefore, the number of pencils in Box A before-change is (25 - 17) units, that is 8 units.
The ratio of the number of pencils in Box A to that in Box B is 8:17 at first.

4. **In Hall A, 30% of the 800 people were men. In Hall B, 40% of the 400 people were women and children. After some of the people in both halls switched halls, 25% of the people in Hall A and 75% of those in Hall B were men. How many people were there in Hall B after the change?**

WORKING

Hall A:

No. of men = $\frac{30}{100}$ x 800 = 240

No. of women + children = 800 – 240 = 560

Hall B:

No. of women + children = $\frac{40}{100}$ x 400 = 160

No. of men = 400 – 160 = 240

Hall A

	M	W&C	Total
Before	240	560	800
Change	?	?	
After	1 unit	3 units	4 units

Hall B

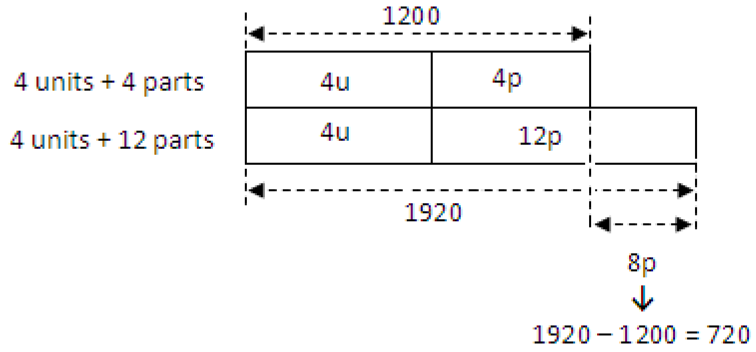
	M	W&C	Total
Before	240	160	400
Change	?	?	
After	3 parts	1 part	4 parts

From Hall A and Hall B's Total column: 4 units + 4 parts → 1200

From Hall A and Hall B's M column: 1 unit + 3 parts → 480

↓ x 4

4 units + 12 parts → 1920



8 parts → 720
1 part → 720 ÷ 8 = 90
4 parts → 90 x 4 = 360

There were <u>360 people</u> in Hall B after the change.

EXPLANATION

Work out all the quantities.

Hall A:

No. of men = $\frac{30}{100}$ x 800 = 240

No. of women + children = 800 – 240 = 560

Hall B:

No. of women + children = $\frac{40}{100}$ x 400 = 160

No. of men = 400 – 160 = 240

Smaller numbers are more manageable.
So, convert the percentage to fraction first.

25% = $\frac{25}{100} = \frac{1}{4}$ 75% = $\frac{75}{100} = \frac{3}{4}$

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

Hall A

	M	W&C	Total
Before	240	560	800
Change	?	?	
After	1 unit	(4 – 1) units ↓ 3 units	4 units

Hall B

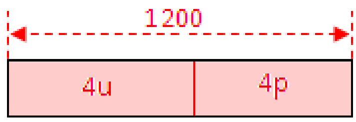
	M	W&C	Total
Before	240	160	400
Change	?	?	
After	3 parts	(4 –3) parts ↓ 1 part	4 parts

CONFUSION ALERT

The percentages are for different situations (Hall A and Hall B).
That means 1 measure in Hall A is different from 1 measure in Hall B.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

Look at the “Total” column for Hall A and Hall B:
Before-change, Hall A had 800 people and Hall B had 400 people.
That’s 1200 people.
After-change, Hall A had 4 units people and Hall B had 4 parts people.
That’s 4 units + 4 parts.
Since the total number of people remains the same before-change and after-change,

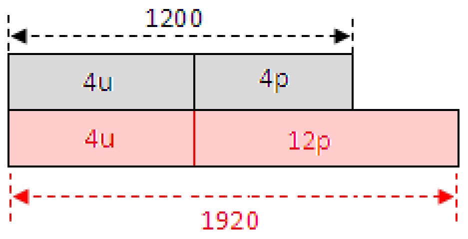
4 units + 4 parts → 1200



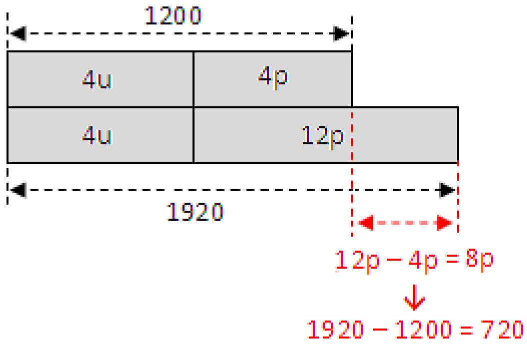
Look at the "Men" column for Hall A and Hall B:
Before-change, Hall A had 240 men and Hall B had 240 men.
That's 480 men.
After-change, Hall A had 1 unit men and Hall B had 3 parts men.
That's 1 unit + 3 parts.
Since the total number of men remains the same before-change and after-change,

1 unit + 3 parts → 480

Multiply this number sentence by 4.
4 units + 12 parts → 1920



Now, calculate the difference.



8 parts → 720
1 part → 720 ÷ 8 = 90
4 parts → 90 x 4 = 360

There were <u>360 people</u> in Hall B after the change.

LET'S APPLY Problems Involving Total Unchanged Quantities

1. Clarice is reading a magazine. The number of pages she has read to the number of pages she has not read is 1:5. If she reads another 100 pages, the number of pages she has read will become 70% of the number of pages she has not read. How many pages are there in the magazine?

2. In a park, the ratio of number birds to squirrels is 8:5. One year later, the number of squirrels increased by 20% and some birds flew away. Given that the total number of birds and squirrels remains the same, find the percentage of the birds that flew away.

3. John's savings was $\frac{5}{13}$ the amount Jessie had in her savings. Jessie then transferred \$600 from her savings to John's savings. As a result, John's savings is now 0.5 times Jessie's savings. What was the total amount of savings John and Jessie had?

4. Tammy had two boxes of erasers. After transferring $\frac{5}{17}$ of the erasers in Box Y to Box X, the number of erasers in Box X becomes 1.5 times the number of erasers in Box Y. What was the ratio of number of erasers in Box X to that in Box Y at first?

5. There are $\frac{5}{8}$ as many chickens as ducks in a farm. The farmer bought more ducks and sold some chickens. As a result, the number of ducks increased by 10%. Given that the total number of chickens and ducks remains the same, find the fraction of the chickens sold.

6. The ratio of number of apples to number of pears in a box was 3:4. When 36 pears were replaced by 36 apples, the ratio of number of apples to number of pears became 3:2. How many apples were there at first?

HIGHER-ORDER PROBLEMS

7. Steve and Tommy had a total of 84 stickers. Steve gave $\frac{1}{3}$ of his stickers to Tommy. Tommy then gave 40% of the total number of stickers he had to Steve. In the end, each of them had an equal number of stickers. How many stickers did each of them have at first?

8. Eugene, Melvin and David had a total of 260 stickers. First, Eugene gave Melvin twice as many stickers as Melvin had. Then Melvin gave David as many stickers as David had. Finally, David gave Eugene as many stickers as Eugene had left. In the end, the ratio of Eugene's to Melvin's to David's stickers was 46:12:7. How many stickers did Eugene have at first?

9. A fruit seller had some red and some green apples. He placed them into two boxes, X and Y. At first, there were 900 apples in Box X and $\frac{3}{10}$ of them were green; and 600 apples in Box Y and $\frac{3}{5}$ of them were green. After he transferred some apples from Box Y to Box X, $\frac{2}{5}$ of the apples in Box X and $\frac{3}{5}$ of the apples in Box Y were green. How many apples were transferred from Box Y to Box X?

1.3 DIFFERENCE UNCHANGED QUANTITIES

DEFINITION

The difference in quantity remains unchanged before-change and after-change.

There are three such situations:

- **Age difference**
  A is 40 years old and B is 10 years old. Their age difference is 30 years.
  4 years later, A will be 44 years old and B will be 14 years old. Their age difference is still 30 years – unchanged.

	A	B	Difference
Before	40	10	30
Change	+4	+4	
After	44	14	30

→ In the A and B change row, the change figure appears in both A and B columns.
The quantities are the same,
and the signs are also the same (+).

↓
A is older than B by 30 years.
Both A and B age by 4 years.
They will never age by different number of years.
So, the change cell is empty (no change occurred).

Although both A and B age by 4 years,
A will still be older than B by 30 years.
Their age difference will always be 30 years.
So, the before-change and after-change Difference remain the same (Difference Unchanged Quantities).

- **A third party gives the individuals involved equal amounts**
  (Amount does not just apply to money but other objects as well)
  A has $40 and B has $100. The difference in their amounts is $60.
  Each of them receives $50 from their father.
  Now A has $90 and B has $150. The difference in their amounts is still $60 – unchanged.

	A	B	Difference
Before	40	100	60
Change	+50	+50	
After	90	150	60

→ In the A and B change row, the change figure appears in both A and B columns.
The quantities are the same,
and the signs are also the same (+).

↓
B has $60 more than A.

Both A and B are given an equal amount ($50).
There is no difference in the amount they are given.
So, the change cell is empty (no change occurred).

Because both A and B are given an equal amount ($50),
B will still have $60 more than A.
So, the before-change and after-change Difference
remain the same (Difference Unchanged Quantities).

- **The individuals involved give away equal amounts**
  Similar to the above situation, except that this time things are subtracted (not added).

EXAMPLES

1. **In the year 2001, Jason was 10 years old and his uncle was 38 years old. In which year was Jason's uncle 8 times as old as Jason?**

WORKING

	Jason's age	His uncle's age	Age difference	Year
Before	10 years	38 years	(38 – 10) years = 28 years	2001
Change	(10 - 4) years ago = 6 years ago	(38 - 32) years ago = 6 years ago		6 years ago
After	1 unit ↓ 1 unit x 4 = 4 years	8 units ↓ 8 units x 4 = 32 years	(8 - 1) units = 7 units ↓ 7 units x 4 = 28 years	(2001 - 6) = 1995

Jason's uncle is 8 times as old as Jason in the year (2001 - 6), that is <u>1995</u>

EXPLANATION

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units.

	Jason's age	His uncle's age	Age difference	Year
Before	10 years	38 years	28 years	2001
Change				
After	1 unit	8 units	(8 - 1) units = 7 units	

We know that their age difference will always be 28 years.
So, we make the age difference after-change (7 units) equal 28 years.
We do this by multiplying the age difference after-change (7 units) by 4.

These actions will convert the after-change row's units to years.

	Jason's age	His uncle's age	Age difference	Year
Before	10 years	38 years	28 years	2001
Change	(10 - 4) years ago = 6 years ago	(38 - 32) years ago = 6 years ago		6 years ago
After	1 unit x 4 = 4 years	8 units x 4 = 32 years	7 units x 4 = 28 years	

Jason's uncle is 8 times as old as Jason (10 - 4) **OR** (38 - 32) years ago,
that is 6 years ago.
Jason's uncle is 8 times as old as Jason in the year (2001 - 6), that is 1995.

2. **Alysia and Benny shared a sum of money in the ratio 2:3. After each of them gave $12 to their brother, the ratio becomes 3:5. Find the amount of money each child had at first.**

WORKING

	Alysia's money	Benny's money	Difference
Before	2 units x 2 = 4 parts	3 units x 2 = 6 parts	1 unit x 2 = 2 parts
Change	- $12 → (4 - 3) parts	- $12 → (6 - 5) parts	
After	3 parts	5 parts	2 parts

1 part → $12
4 parts → ($12 x 4) = $48
6 parts → ($12 x 6) = $72

Alysia had 4 parts, which is equivalent to $48, and
Benny had 6 parts, which is equivalent to $72 at first.

EXPLANATION

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Alysia's money	Benny's money	Difference
Before	2 units	3 units	1 unit
Change	- $12	- $12	
After	3 parts	5 parts	2 parts

CONFUSION ALERT

The 2 ratios are for different situations (before-change and after-change).
That means 1 measure in the first ratio is different from 1 measure in the second ratio.
Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know the difference in their share of money remains unchanged.
So, we make the difference in their share of money after-change (2 parts) equal that before-change (1 unit).
We do this by multiplying the difference in their share of money before-change (1 unit) by 2.

These actions will convert the before-change row's units to parts.

	Alysia's money	Benny's money	Difference
Before	2 units x 2 = 4 parts	3 units x 2 = 6 parts	1 unit x 2 = 2 parts
Change	- $12 → (4 - 3) parts	- $12 → (6 - 5) parts	
After	3 parts	5 parts	2 parts

The change in each of their share of money is (4 - 3) **OR** (6 - 5) part, that is 1 part, which is equivalent to $12.
1 part → $12
4 parts → ($12 x 4) = $48
6 parts → ($12 x 6) = $72
Alysia had 4 parts, which is equivalent to $48, and
Benny had 6 parts, which is equivalent to $72 at first.

3. **Last year, the number of people who visited the zoo was 2700 and number of people who visited the bird park was 8800. This year, the number of people visiting both places decreased by the same number. Given that the number of people who visited the bird park is 6 times that who visited the zoo this year, what was the decrease in the number of people to each place?**

WORKING

	Visitors to the zoo	**Visitors to the bird park**	**Difference**
Before	2700 people	8800 people	(8800 – 2700) people = 6100 people
Change	(2700 - 1220) less people = 1480 less people	(8800 - 7320) less people = 1480 less people	
After	1 unit → (6100 ÷ 5) people = 1220 people	6 units → (1220 x 6) people = 7320 people	5 units → 6100 people

The decrease in the number of people to each place is (2700 - 1220) **OR** (8800 - 7320) people, that is 1480 people.

EXPLANATION

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units.

	Visitors to the zoo	Visitors to the bird park	Difference
Before	2700 people	8800 people	(8800 - 2700) people = 6100 people
Change	- ? people	- ? people	
After	1 unit	6 units	(6 - 1) units = 5 units

We know the difference between the number of visitors to the zoo and to the bird park remains unchanged.
So, we make the difference after-change (5 units) equal that before-change (6100 people).
We do this by multiplying the difference after-change (5 units) by 1220.

These actions will convert the after-change row's units to people.

	Visitors to the zoo	Visitors to the bird park	Difference
Before	2700 people	8800 people	6100 people
Change	- ? people	- ? people	
After	1 unit x 1220 = 1220 people	6 units x 1220 = 7320 people	5 units x 1220 = 6100 people

The decrease in the number of people to each place is (2700 - 1220) **OR** (8800 - 7320) people, that is 1480 people.

LET'S APPLY Problems Involving Difference Unchanged Quantities

1. A container contains some red and some green marbles. At first, the number of red marbles was 30% that of the number of green marbles. After adding 75 marbles of each colour, the number of red marbles becomes 80% that of the green marbles. How many marbles of each colour were there at first?

2. Rose's salary is $\frac{7}{11}$ of her husband's. Each of them saves $390 and spends the rest of their money. If Rose's expenditure to that of her husband's is in the ratio of 1:2, find the salary of each person.

3. A group of children visited the Botanic Gardens one morning. The ratio of the number of boys to the number of girls was 3:2. Later in the afternoon, 12 boys and 12 girls joined the group. The new ratio became 19:14. Find the total number of children in the Botanic Gardens that afternoon.

4. Uncle John is 24 years older than his nephew. His age will be 3 times his nephew's in 6 years' time. Find the sum of their present ages.

5. Harry had $400 less than Linden at first. When they both spent their money to buy the same wallet, the ratio of Linden's money to Harry's money became 3:1. Given that the total amount of money they had left is 2.5 times the cost of the wallet, find the cost of the wallet.

HIGHER-ORDER PROBLEMS

6. Three brothers, Andy, Benny and Calvin shared some money in the ratio of 6:5:1. After their mother gave each of them $27, the ratio became 15:13:5. Find the amount of money each of the boys had at first.

7. The ratio of Sally's age to Gillian's age is 2:7. In 15 years' time, the ratio of Sally's age to Gillian's age will become 1:2. In how many years' time will their combined age be 70 years old?

8. Adeline's monthly salary is $350 more than Bernice's. Each of them spends $800 per month. After some time, Adeline saves $1950 while Bernice saves only $900.
   a) How long does each of them take to save the given amount of money?
   b) What is Adeline's monthly salary?

9. Sam bought some sweets and chocolates. When he gave 30 sweets and 30 chocolates to his friends, he had $\frac{2}{5}$ as many sweets as chocolates. He then bought another 5 sweets and 5 chocolates. As a result, 30% of the candies were sweets. How many sweets did he buy at first?

Answers to questions in the prior chapters' Let's Apply sections
are listed in this chapter. Detailed workings may be downloaded at:

www.mathsheuristics.com/solutions

CHAPTER 1 : SINGLE UNCHANGED QUANTITIES

LET'S APPLY Problems Involving Single Unchanged Quantities

1. 120 children

2. 144 more seashells than pebbles

3. 90 sparrows

4. a) 72 apple pies
   b) 24 cakes

5. 75 VIP seats

HIGHER-ORDER PROBLEMS

6. 200 sweets

7. 148 candies

8. 68 more Singapore stamps than Malaysia stamps

9. $40

CHAPTER 2 : TOTAL UNCHANGED QUANTITIES

LET'S APPLY Problems Involving Total Unchanged Quantities

1. 408 pages

2. 12.5%

3. $10800

4. 13:17

5. $\frac{4}{25}$

6. 90 apples

HIGHER-ORDER PROBLEMS

7. Steve had 21 stickers
   Tom had 63 stickers

8. 164 stickers

9. 450 apples

CHAPTER 3 DIFFERENCE UNCHANGED QUANTITIES

LET'S APPLY Problems Involving Difference Unchanged Quantities

1. 9 red marbles
   30 green marbles

2. Rose's salary is $910
   Her husband's salary is $1430

3. 99 children

4. 36 years old

5. $320

HIGHER-ORDER PROBLEMS

6. Andy had $108
   Benny had $90
   Calvin had $18

7. 12.5 years

8. (a) 3 months
   (b) $1450

9. 70 sweets

Mastering Heuristics Series
Handbooks for discerning parents

About the Book

Heuristics is a specialised problem-solving concept now taught at primary-level maths. Merged into the regular maths curriculum, it is difficult to isolate and learn, making maths confusing to some. Many parents must set aside the regular-syllabus they learned in their youth to re-learn and teach their children heuristics – much easier said than done!

To give parents and students a complete and comprehensive guide to heuristics, Sunny Tan, Principle Trainer of mathsHeuristics™, wrote this series of four books. The series neatly packages heuristics by concepts (series of books) and various well-defined application scenarios (chapters in each book). It also offers many examples, showing the efficiency of – and step-by-step application of – heuristics techniques.

This book specifically teaches the Unit Transfer Method – the use of ratio to effectively analyse and solve challenging maths problems. This simple, logical yet powerful problem-solving technique is an alternative to the model and algebraic approaches.

While each book introduces parents to a particular heuristics concept under various scenarios, it gives students the opportunity to see how the specific heuristics technique works, and to get in some practice. For students enrolled in mathsHeuristics™ programmes, each book serves as a study companion, while keeping parents well-informed of what their children are learning.

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New Features in 2nd edition:

- Further refined UTM techniques to solve wider range of questions
- Higher-order thinking questions value-add to the learning
- Complementary web-based interface for live discussions and UTM demonstrations

About the Author

Sunny Tan currently trains students in the application of various heuristics concepts, with special focus on students in their critical year – the PSLE year. He also conducts heuristics workshops for parents and educators.

For over 10 years in the 90s, NIE-trained Sunny taught primary and secondary maths in various streams. He observed how the transformed primary maths syllabus stumped children, parents and, sometimes, even teachers. How do you teach young children to accurately choose and sequentially apply different situational logic in solving non-routine problems?

Sunny resolved to simplify the learning and application of such skills. Through years of research and development, Sunny eventually established the mathsHeuristics™ programme. Result-oriented research has since proven the consistent effectiveness of the mathsHeuristics™ programme.

Sunny's ingenious methodology has attracted much media interest – ParentsWorld, Wawa Magazine, The Straits Times, KiasuParents.com, Absolutely Parents, TODAY, The Singapore's Child – as well as raving reviews by academia and parents.

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