Mastering Heuristics Series
Handbooks for discerning parents

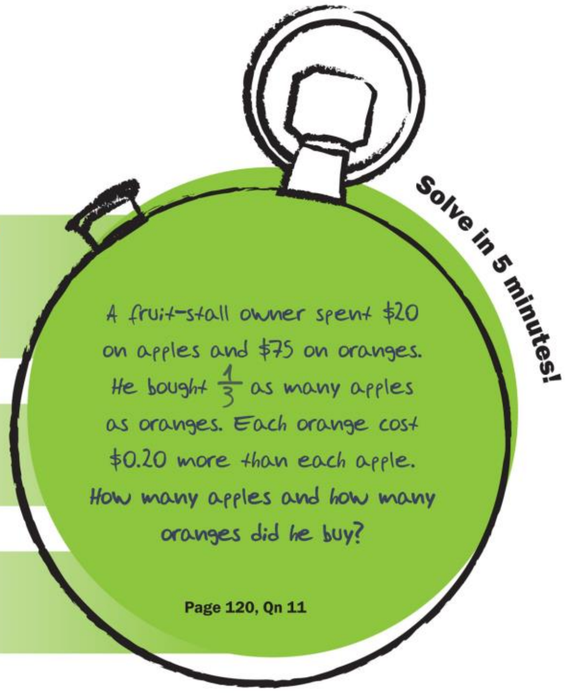
1:UNIT TRANSFER METHOD

A Problem-solving Tool

for Challenging Problems in Upper Primary Mathematics

Sunny Tan

Primary 4



Mastering Heuristics Series
Handbook for discerning parents

Unit Transfer Method
A Problem-solving Tool
for Challenging Problems
in Upper Primary Mathematics
(Primary 4)

Sunny Tan
Maths Heuristics (S) Private Limited

ISBN: 978-981-08-8148-1

First Published 2011
Printed in Singapore 2011

Published by Maths Heuristics (S) Private Limited
Edited by Karen Ralls-Tan of RE: TEAM Communications
Distributed by **Maths Heuristics (S) Private Limited**

195A Thomson Road, Goldhill Centre, Singapore 307634
www.mathsHeuristics.com
Tel: 6893 5593

For orders and enquiries:

Email: enquiry@mathsheuristics.com

CONTENTS

PREFACE

This book teaches Unit Transfer Method at Primary 4, which is the use of proportion to effectively analyse and solve challenging mathematical problems involving whole numbers, fractions and decimals. It delivers the foundation for learning Unit Transfer Method at Primary 5 where mathematical problems are expanded to involve ratios and percentages.

Ultimately, Unit Transfer Method is a simple, logical yet powerful problem-solving technique that complements the model approach and the algebraic approach. It also paves the way for students to eventually learn the algebraic approach in secondary school.

Heuristics in Primary Maths Syllabus

Heuristics is a specialised mathematical problem-solving concept. Mastering it facilitates efficiency in solving regular as well as challenging mathematical problems. The Ministry of Education in Singapore has incorporated 11 Problem-Solving Heuristics into all primary-level mathematical syllabus.

Learning Heuristics Effectively

Instead of containing the 11 Problem-Solving Heuristics neatly into specific chapters though, they have been integrated into the regular curriculum. This not only makes it difficult for students to pick up Heuristics skills, but can also make mathematics confusing for some students. For us parents, it is difficult for us to put aside the regular-syllabus mathematical concepts we were brought up on to re-learn Heuristics, much less teach our own children this new concept.

Take Algebraic Equations, for instance. Although it is a Heuristics technique, the topic has never been, and is still not, taught at primary level. Yet, primary-level Mathematics Papers these days include questions from the topic. Parents, being familiar with the topic, will attempt to teach their children to solve the questions using Algebraic Equations. This will only confuse their children. According to current primary-level mathematical syllabus, other Heuristics techniques should be used instead.

These and other challenges were what I observed first-hand during my years as a mathematics teacher, and what provided me the impetus for my post-graduate studies, mathsHeuristics™ programmes and, now, the Mastering Heuristics Series of books.

mathsHeuristics™ programmes and the Mastering Heuristics Series of books cater to different levels, from Primary 4 to Primary 6. Students with a firm foundation in heuristics at Primary 4 will experience a smoother transition to Primary 5 and Primary 6 mathematics. The Primary 5 and Primary 6 mathematical syllabi no longer aim to teach heuristics techniques, focusing instead on technique application in problem-solving situations. As such, it is important for Primary 4 students to become familiar and comfortable with heuristics techniques before they move up to Primary 5.

About Mastering Heuristics Series

This series of books is a culmination of my systematic thinking, supported by professional instructional writing and editing, to facilitate understanding and mastery of Heuristics. I would also like to acknowledge the support from two individuals in local academia – Mr Ammiel Wan, an authority in local mathematics syllabus, who shared his knowledge in and experience with the teaching of heuristics. Dr Lua Eng Keong, an overseas academic and a Singapore parent, who is now back in Singapore. Through his then 9-year-old son, feedback and suggestions, Dr Lua provided a test bed and refinement opportunities for the methodologies used to teach heuristics in the Mastering Heuristics Series.

Through this series, I have neatly packaged Heuristics into different techniques (Series of books) and application scenarios (Chapters within each book). For each application scenario, I offer many examples, showing how the technique may be applied, and then explaining the application in easy-to-follow steps and visualisations without skipping a beat.

The entire series of books provides a complete and comprehensive guide to Heuristics.

Multi Benefits to Students, Parents and Educators

While each book introduces parents to one Heuristics technique applied across diverse scenarios, it gives students the opportunity to see how the specific Heuristics technique works as well as get in some practice.

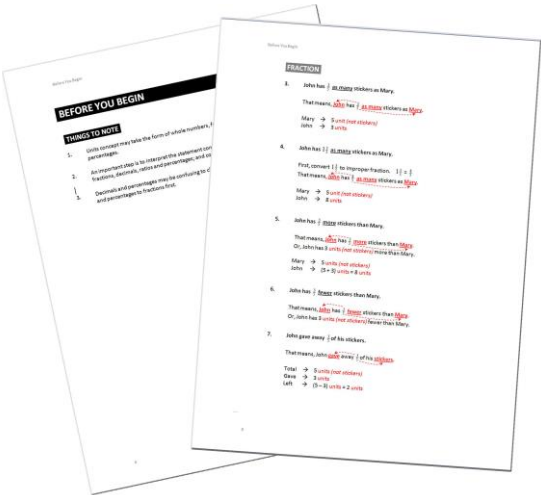
For students enrolled in mathsHeuristics™ programmes, each book serves as a great companion, while keeping parents well-informed of what their children are learning.

Sunny Tan

August 2011

HOW TO USE THIS BOOK

BEFORE YOU BEGIN

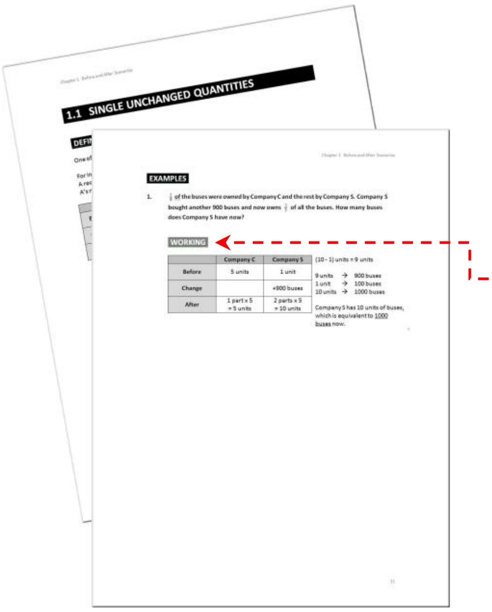


The “Before You Begin” chapter instills the basic but important step that must be applied across every question under this topic. This helps to standardise the given information for easy application of the techniques being taught.

In this book on Unit Transfer Method (UTM) at Primary 4, this step is to convert whole numbers, fractions and decimals into units.

CHAPTERS AND SECTIONS

The various UTM techniques are neatly separated into different chapters and sections. Thus, examples of UTM application are classified according to problem-solving techniques for more focused learning.

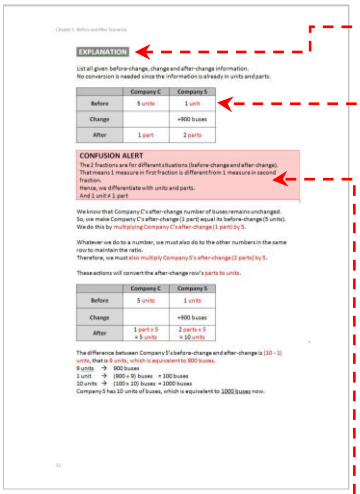


EXAMPLES

Each example of UTM application comes with “Working” and “Explanation”, which includes “Confusion Alert” boxes.

WORKING

“Working” shows heuristics application in action (how quick it is to solve a question).



EXPLANATION

"Explanation" shows the thought process behind the heuristics application (the detailed steps). It takes readers through the solution in the following manner:

- step-by-step without skipping a beat so that readers can follow what happens at each and every step.
- systematic so that readers begin to see a pattern in applying the technique.
- easy-to-follow so that readers can quickly understand the technique minus the frustration.

In UTM, readers will see that its application always begins with:

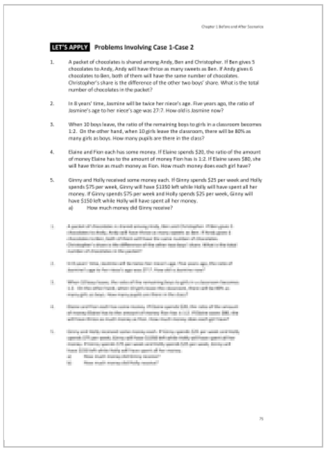
- the basic step explained in "Before You Begin", and
- the tabulation of all given information.

This quickly helps students see and understand the relationships among all the information given in the question.

"Confusion Alert" boxes in the midst of "Explanation" highlight areas where students are likely to be uncertain of or make mistakes in. It also gives the rationale to help clarify doubts in these areas.

LET'S APPLY

Learning is only effective when followed up with practice. Hence, at the end of each chapter/section is a list of questions related to the heuristics technique taught in that chapter/section.



ADDITIONAL TIPS

For on-going sharing and discussions on the use of UTM, visit:
www.unittransfermethod.blogspot.com

For detailed workings to all UTM "Let's Apply" section, visit:
www.mathsHeuristics.com/?page_id=472

BEFORE YOU BEGIN

THINGS TO NOTE

1. The units concept may take the form of whole numbers, fractions and decimals.

2. An important step is to interpret the statement containing whole numbers, fractions and decimals; and convert them into units.

3. Decimals may be confusing to children. So, convert any decimals to fractions first.

EXAMPLES

WHOLE NUMBER

1. **John has 5 times <u>as many</u> stickers as Mary.**

First, convert the whole number to fraction. $5 = \frac{5}{1}$

That means, <u>John</u> has $\frac{5}{1}$ times <u>as many</u> stickers as <u>Mary</u>.

Mary → 1 unit *(not stickers)*
John → 5 units

2. **John has 5 times <u>more</u> stickers than Mary.**

First, convert the whole number to fraction. $5 = \frac{5}{1}$

That means, <u>John</u> has $\frac{5}{1}$ times <u>more</u> stickers than <u>Mary</u>.

Mary → 1 unit *(not stickers)*
John → (1 + 5) units = 6 units

FRACTIONS

3. **John has $\frac{3}{5}$ as many stickers as Mary.**

That means, **John** has $\frac{3}{5}$ **as many** stickers as **Mary**.

Mary → 5 units *(not stickers)*
John → 3 units

4. **John has $1\frac{3}{5}$ as many stickers as Mary.**

First, convert $1\frac{3}{5}$ to improper fraction. $1\frac{3}{5} = \frac{8}{5}$
That means, **John** has $\frac{8}{5}$ **as many** stickers as **Mary**.

Mary → 5 units *(not stickers)*
John → 8 units

5. **John has $\frac{3}{5}$ more stickers than Mary.**

That means, **John** has $\frac{3}{5}$ **more** stickers than **Mary**.
Or, John has 3 units *(not stickers)* more than Mary.

Mary → 5 units *(not stickers)*
John → (5 + 3) units = 8 units

6. **John has $\frac{3}{5}$ fewer stickers than Mary.**

That means, **John** has $\frac{3}{5}$ **fewer** stickers than **Mary**.
Or, John has 3 units *(not stickers)* fewer than Mary.

Mary → 5 units *(not stickers)*
John → (5 – 3) units = 2 units

7. **John gave away $\frac{3}{5}$ of his stickers.**

That means, John **gave** away $\frac{3}{5}$ of his **stickers**.

Total → 5 units *(not stickers)*
Gave → 3 units
Left → (5 – 3) units = 2 units

DECIMALS

8. John has 0.6 times <u>as many</u> stickers as Mary.

First, convert the decimal to fraction. $0.6 = \frac{6}{10} = \frac{3}{5}$

That means, **<u>John</u>** has $\frac{3}{5}$ **<u>as many</u>** stickers as **<u>Mary</u>**. **(Just like Example 3)**

Mary → 5 units *(not stickers)*
John → 3 units

9. John has 1.6 times <u>as many</u> stickers as Mary.

First, convert the decimal to improper fraction. $1.6 = 1\frac{6}{10} = 1\frac{3}{5} = \frac{8}{5}$

That means, **<u>John</u>** has $\frac{8}{5}$ **<u>as many</u>** stickers as **<u>Mary</u>**. **(Just like Example 4)**

Mary → 5 units *(not stickers)*
John → 8 units

10. John has 0.6 times <u>more</u> stickers than Mary.

First, convert the decimal to fraction. $0.6 = \frac{6}{10} = \frac{3}{5}$

That means, **<u>John</u>** has $\frac{3}{5}$ **<u>more</u>** stickers than **<u>Mary</u>**. **(Just like Example 5)**

Or, John has 3 units *(not stickers)* more than Mary.

Mary → 5 units *(not stickers)*
John → (5 + 3) units = 8 units

11. John has 0.6 times <u>fewer</u> stickers than Mary.

First, convert the decimal to fraction. $0.6 = \frac{6}{10} = \frac{3}{5}$

That means, **<u>John</u>** has $\frac{3}{5}$ **<u>fewer</u>** stickers than **<u>Mary</u>**. **(Just like Example6)**

Or, John has 3 units *(not stickers)* fewer than Mary.

Mary → 5 units *(not stickers)*
John → (5 – 3) units = 2 units

12. John gave away 0.6 of his stickers.

First, convert the decimal to fraction. $0.6 = \frac{6}{10} = \frac{3}{5}$

That means, John **<u>gave</u>** away $\frac{3}{5}$ of his **<u>stickers</u>**. **(Just like Example 7)**

Total → 5 units *(not stickers)*
Gave → 3 units
Left → (5 – 3) units = 2 units

CHAPTER 1 BEFORE AND AFTER SCENARIOS

STEPS

- List all given before-action, change-action and after-action information.
- Convert the information into units and parts,
  where necessary and if not already in units and parts.
- Compare the information to find the unknown.

APPLICABILITY

There are four basic scenarios where the Before and After Concept may be applied.

- Single Unchanged Quantities
- Total Unchanged Quantities
- Difference Unchanged Quantities
- All Changing Quantities

1.1 SINGLE UNCHANGED QUANTITIES

DEFINITION

One of the given quantities remains unchanged.

ILLUSTRATION

For instance, A and B have stickers in a certain quantities (Say, 20 and 50 stickers).
A receives 5 stickers from C (external party).
A's number of stickers changed, B's number of stickers remains unchanged.

	A	B
Before	20	50
Change	+5	
After	25	50

→ In the change row, the change figure appears in the column where the change occurred.

↓

In the column where the change cell is empty (no change occurred), the before-change and after-change quantities remain the same (quantities unchanged).

While A's number of stickers changes (+5), B's number of stickers remains unchanged (Single Unchanged Quantity).

EXAMPLES

1. **Wilson had 4 times as many fish as Ian. After Ian bought 3 additional fish from the pet shop, Ian had 0.5 times as many fish as Wilson. How many fish did Wilson have at first?**

WORKING

	Wilson's fish	**Ian's fish**
Before	4 units	1 unit
Change		+3 fish
After	2 parts x 2 = 4 units	1 part x 2 = 2 units

(2 – 1) units = 1 unit

1 unit → 3 fish
4 units → (4 x 3) fish = 12 fish

Wilson had 12 fish at first.

EXPLANATION

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Wilson's fish	Ian's fish
Before	4 units	1 unit
Change		+ 3 fish
After	2 parts	1 part

CONFUSION ALERT

The ratio and decimal are for different situations (before-change and after-change).
That means 1 measure in first whole number is different from 1 measure in second decimal. Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that Wilson's after-change number of fish remains unchanged.
So, we make Wilson's after-change (2 parts) equal its before-change (4 units).
We do this by multiplying Wilson's after-change (2 parts) by 2.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must also multiply Ian's after-change (1 part) by 2.

These actions will convert the after-change row's parts to units.

	Wilson's fish	Ian's fish
Before	4 units	1 unit
Change		+3 fish
After	2 parts x 2 = 4 units	1 part x 2 = 2 units

The difference between Ian's before-change and after-change is (2 – 1) unit, that is 1 unit.
We know that Ian bought 3 additional fish from the pet shop.
1 unit → 3 fish

Wilson had 4 units of fish at first.
4 units → (4 x 3) fish = 12 fish

Wilson had 12 fish at first.

2. **The number of women at a zoo was $\frac{7}{10}$ the number of men. After 121 women left the zoo, the number of women remaining became $\frac{1}{3}$ the number of men remaining. How many women were there at the zoo at first?**

WORKING

	Men	**Women**
Before	10 units x 3 = 30 units	7 units x 3 = 21 units
Change		– 121 women
After	3 parts x 10 = 30 units	1 part x 10 = 10 units

(21 – 10) units = 11 units

11 units → 121 women
1 unit → (121 ÷ 11) women = 11 women
21 units → (21 x 11) women = 231 women

There were 231 women at the zoo at first.

EXPLANATION

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Men	Women
Before	10 units	7 units
Change		– 121 women
After	3 parts	1 part

CONFUSION ALERT
The 2 fractions are for different situations (before-change and after-change).
That means 1 measure in first fraction is different from 1 measure in second fraction. Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that the number of men remains unchanged.
So, we make the men's after-change (3 parts) equal their before-change (10 units).
We do this by multiplying the men's after-change (3 parts) by 10,
and their before-change (10 units) by 3.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must also multiply the women's after-change (1 part) by 10,
and their before-change (7 units) by 3.

These actions will convert the after-change row's parts to units.

	Men	Women
Before	10 units x 3 = 30 units	7 units x 3 = 21 units
Change		– 121 women
After	3 parts x 10 = 30 units	1 part x 10 = 10 units

The difference between the women's before-change and after-change is (21 – 10) units, that is 11 units.
We know that 121 women left the zoo.
11 units → 121 women
1 unit → (121 ÷ 11) women = 11 women

There were 21 units of women at the zoo at first.
21 units → (21 x 11) men = 231 women

There were 231 women at the zoo at first.

3. **Colin had 0.7 times as many stamps as Daniel. After Daniel gave away 54 stamps to his friends, both boys had an equal number of stamps left. How many stamps did Daniel have at first?**

WORKING

0.7 = $\frac{7}{10}$

	Colin's stamps	Daniel's stamps
Before	7 units	10 units
Change		– 54 stamps
After	1 part x 7 = 7 units	1 part x 7 = 7 units

(10 – 7) units = 3 units

3 units → 54 stamps
1 unit → (54 ÷ 3) stamps = 18 stamps
10 units → (10 x 18) stamps = 180 stamps

Daniel had 180 stamps at first.

EXPLANATION

Convert the decimal to fraction first.
0.7 = $\frac{7}{10}$

List all given before-change, change and after-change information.
No conversion is needed since the information is already in units and parts.

	Colin's stamps	Daniel's stamps
Before	7 units	10 units
Change		– 54 stamps
After	1 part	1 part

CONFUSION ALERT
The decimal and ratio are for different situations (before-change and after-change). That means 1 measure in first decimal is different from 1 measure in second whole number. Hence, we differentiate with units and parts.
And 1 unit ≠ 1 part

We know that the number of Colin's stamps remains unchanged.
So, we make the Colin's stamps' after-change (1 part) equal its before-change (7 units).
We do this by multiplying Colin's after-change (1 part) by 7,
and his before-change (7 units) by 1.

Whatever we do to a number, we must also do to the other numbers in the same row to maintain the ratio.
Therefore, we must also multiply Daniel's after-change (1 part) by 7,
and his before-change (10 units) by 1.

These actions will convert the after-change row's parts to units.

	Colin's stamps	**Daniel's stamps**
Before	7 units x 1 = 7 units	10 units x 1 = 10 units
Change		– 54 stamps
After	1 part x 7 = 7 units	1 part x 7 = 7 units

The difference between Daniel's before-change and after-change is (10 - 7) units, that is 3 units.
We know that Daniel gave away 54 stamps.
3 units → 54 stamps
1 unit → (54 ÷ 3) stamps = 18 stamps

Daniel had 10 units of stamps at first.
10 units → (10 x 18) stamps = 180 stamps

Daniel had 180 stamps at first.

LET'S APPLY Problems Involving Single Unchanged Quantities

1. Tarzan had twice as many monkeys as Jane. After 120 of Tarzan's monkeys ran away, Jane had thrice as many monkeys as Tarzan. How many monkeys did Tarzan have at first?

2. Eve had twice as many apples as Adam. After Adam gave 8 of his apples to his children, Adam had $\frac{1}{3}$ as many apples as Eve. How many apples did Eve and Adam have altogether at first?

3. Lily had 3 times as many pairs of earrings as necklaces. After giving away 18 necklaces, she had 5 times as many pairs of earrings as necklaces. How many pairs of earrings did she have?

4. Daisy had 3 times as many candies as Rose. After Daisy gave 6 candies to her sister, she had 1.5 times as many candies as Rose. How many candies did Rose have?

5. $\frac{7}{13}$ of Grace's beads are red and the rest are purple. If she gives 15 red beads away, she will have an equal number of red beads and purple beads. How many purple beads does she have?

6. Benny and Tom had the same number of stamps. When Benny gave away 147 of his stamps, Benny had 0.25 times as many stamps as Tom. How many stamps did the two boys have altogether at first?

7. There were some marbles and 42 rubber balls in a box. Jessica took out 20 marbles from the box. As a result, there were twice as many rubber balls as marbles in the box. How many marbles were there in the box at first?

8. There were thrice as many children as adults in a hall. After 7 adults left the hall, there were 6 times as many children as adults in the hall. How many people were there in the hall at first?

9. Janice had 5 times as many boxes of chocolates as Jarrod. If Jarrod gave away 6 boxes of chocolates to his sister, he would have $\frac{1}{8}$ as many boxes of chocolates as Janice.
   a) How many boxes of chocolates did Janice have?
   b) If each box contained 12 pieces of chocolates, how many pieces of chocolates does Janice have in all?

10. Pauline has $\frac{1}{8}$ times as many red packets as Esther. Pauline receives 9 more red packets. Now, she has 0.5 times as many red packets as Esther. How many red packets does Esther have?

Answers to questions in the prior chapters' Let's Apply sections
are listed in this chapter. Detailed workings may be downloaded at:

www.mathsheuristics.com/solutions

CHAPTER 1 SINGLE UNCHANGED QUANTITIES

1. 144 monkeys
2. 72 apples
3. 135 pairs of earrings
4. 4 candies
5. 90 purple beads
6. 392 stamps
7. 41 marbles
8. 56 people
9. a) 80 boxes

   b) 960 pieces
10. 24 red packets