Mastering Heuristics Series
Handbooks for discerning parents

Sam won some money in a lottery. He saved 25% of his winnings and gave his parents $1500 less than the amount he saved. He spent 40% more on a computer than what he saved. Given that he had $3630 left, find the amount of money he won in the lottery.

Parents
Computer

Primary 5 & 6

MODEL APPROACH to Problem-solving

Stack & Split to solve challenging problems fast!

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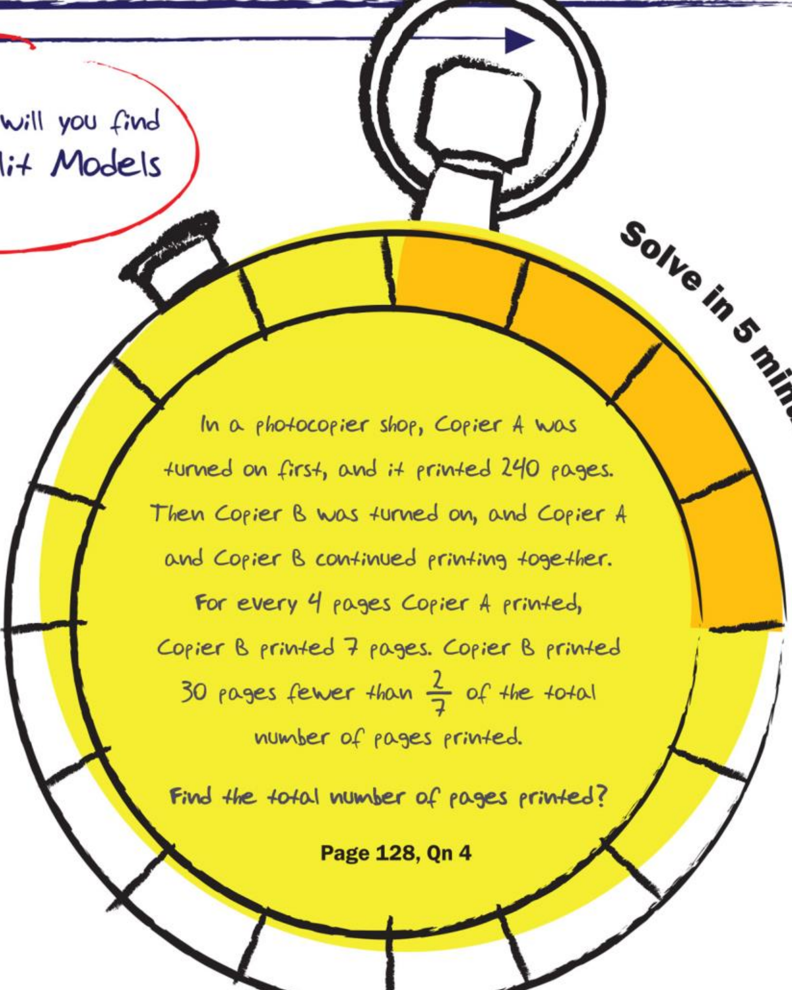

Sunny Tan

Mastering Heuristics Series
Handbook for discerning parents

Model Approach to Problem-solving
(Primary 5 & 6)

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Maths Heuristics (S) Private Limited

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ANSWERS

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PART 2 STACK MODEL

PART 3 SPLIT MODEL

PREFACE

This book teaches the Model Approach to Problem-solving (MAPS), which is the use of drawings to effectively analyse and solve challenging mathematical problems. This highly-visual problem-solving technique complements the Unit Transfer Method. It also paves the way for students to eventually learn the algebraic approach in secondary school. Most importantly, it is a Heuristics technique that is most commonly emphasised in school.

Heuristics in Primary Maths Syllabus

Heuristics is a specialised mathematical problem-solving concept. Mastering it facilitates efficiency in solving regular as well as challenging mathematical problems. The Ministry of Education in Singapore has incorporated 11 Problem-Solving Heuristics into all primary-level mathematical syllabus.

Learning Heuristics Effectively

However, instead of containing the 11 Problem-Solving Heuristics neatly into specific chapters, they have been integrated into the regular curriculum. This not only makes it difficult for students to pick up Heuristics skills, but can also make mathematics confusing for some students. For us parents, it is difficult to put aside the mathematical concepts in the regular syllabus that we were brought up on in order to re-learn Heuristics, much less teach our own children this new concept.

Take Algebraic Equations, for instance. Although it is a Heuristics technique, the topic has never been, and is still not, taught at primary level. Yet, primary-level Mathematics Papers these days include questions from the topic. Parents, being familiar with the topic, will attempt to teach their children to solve the question using Algebraic Equations. This will only confuse their children. According to current primary-level mathematical syllabus, other Heuristics techniques should be used instead.

These and other challenges were what I observed first-hand during my years as a mathematics teacher; and what provided me the impetus for my post-graduate studies, mathsHeuristics™ programmes and, now, the Mastering Heuristics Series of books.

About Mastering Heuristics Series

This series of books is a culmination of my systematic thinking, supported by professional instructional writing and editing, to facilitate understanding and mastery of Heuristics. Through it, I have neatly packaged Heuristics into different techniques (Series of books) and application scenarios (Chapters within each book). For each application scenario, I offer many examples, showing how the technique may be applied, and then explaining the application in easy-to-follow steps without skipping a beat.

The entire series of books provides a complete and comprehensive guide to Heuristics.

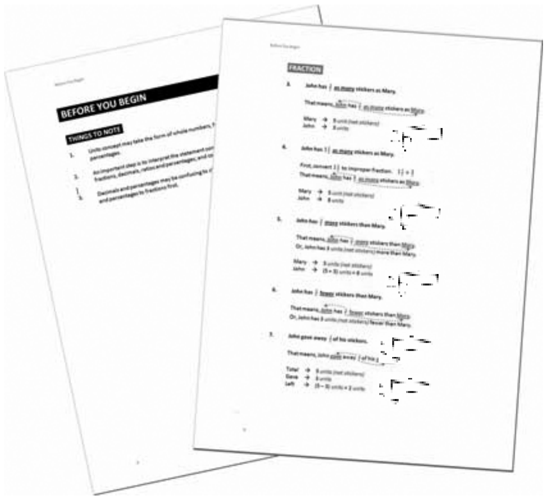
While each book introduces parents to one Heuristics technique applied across diverse scenarios, it gives students the opportunity to see how the specific Heuristics technique works as well as get in some practice. For students enrolled in mathsHeuristics™ programmes, each book serves as a great companion, while keeping parents well-informed of what their children are learning.

Sunny Tan

November 2010

HOW TO USE THIS BOOK

BEFORE YOU BEGIN

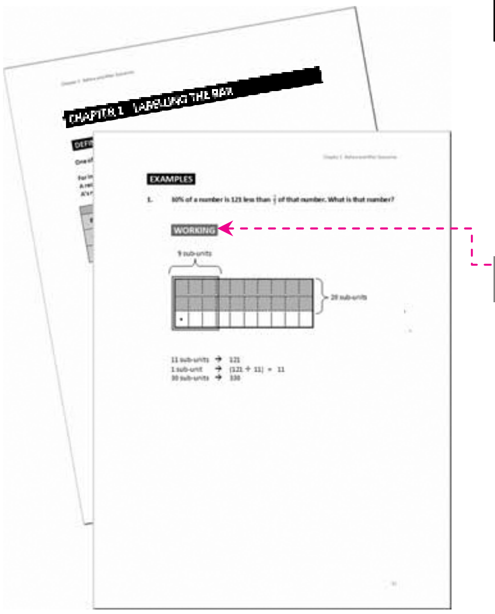


The "Before You Begin" chapters instill basic but important steps that must be applied across every question when using this Heuristics technique. This helps to standardise the given information for easy application of the technique being taught.

In this book on the Model Approach to Problem-solving (MAPS),
this step is to convert whole numbers, fractions, decimals, percentages and ratios into units; and then to present the converted information in drawings.

PARTS AND CHAPTERS

The MAPS technique can be applied in different ways. These are neatly separated into different parts within the book. The MAPS technique can also be applied to different scenarios. These are packaged as separate chapters within the book. Thus, examples of MAPS application are classified according to ways of usage and problem scenarios, for more focused learning.

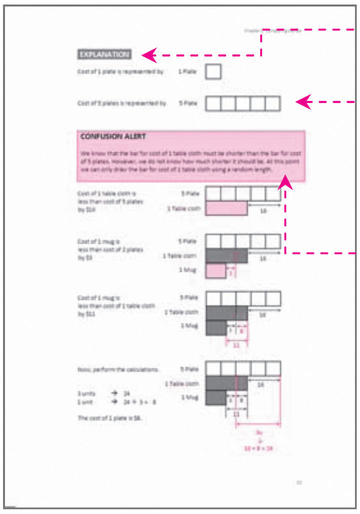


EXAMPLES

Each example of MAPS application comes with "Working" and "Explanation", which includes "Confusion Alert" boxes.

WORKING

"Working" shows Heuristics application in action (how quick it is to solve a question).



EXPLANATION

"Explanation" shows the thought process behind the Heuristics application (the detailed steps). It takes readers through the solution in the following manner:

- step-by-step without skipping a beat so that readers can follow what happens at each and every move.
- systematic so that readers begin to see a pattern in applying the technique.
- easy-to-follow so that readers can quickly understand the technique minus the frustration.

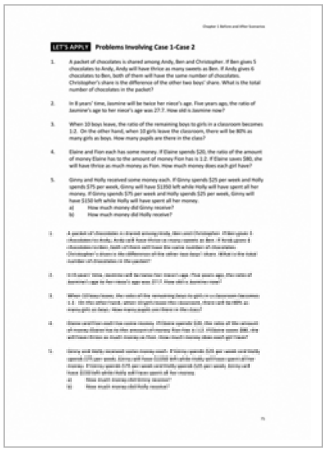
In MAPS, readers will see that its application always begins with:

- the basic step explained in "Before You Begin", and
- the presentation of all given information in a model.

This quickly helps students see and understand the relationships among all the information given in the question.

"Confusion Alert" boxes in the midst of "Explanation" highlight areas where students are likely to be uncertain of or make mistakes in. It also gives the rationale to help clarify doubts in these areas.

LET'S APPLY



Learning is only effective when followed up with practice. Hence, at the end of each chapter/section is a list of questions related to the Heuristics technique taught in that chapter/section.

ADDITIONAL TIPS

For on-going sharing and discussions on the use of MAPS, visit:
www.unittransfermethod.blog.com

For detailed workings to all MAPS "Let's Apply" sections, visit:
www.mathsheuristics.com/?page_id=472

BEFORE YOU BEGIN

THINGS TO NOTE

1. Model concept may take the form of whole numbers, fractions, decimals, ratios or percentages.

2. An important step is to interpret the statement containing whole numbers, fractions, decimals, ratios and percentages; and convert them into units.

3. Decimals and percentages may be confusing to children. So, convert any decimals and percentages to fractions first.

4. Reduce fractions to lowest term. Smaller numbers are easier for children to manage.

EXAMPLES

WHOLE NUMBER

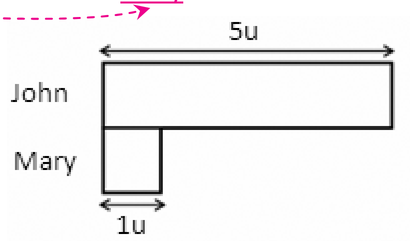
1. **John has 5 times <u>as many</u> stickers as Mary.**

First, convert the whole number to fraction. 5 = $\frac{5}{1}$

That means, John has $\frac{5}{1}$ times as many stickers as Mary.

Mary → 1 unit *(not stickers)*

John → 5 units



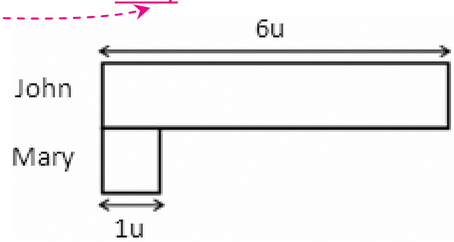
2. **John has 5 times <u>more</u> stickers than Mary.**

First, convert the whole number to fraction. 5 = $\frac{5}{1}$

That means, John has $\frac{5}{1}$ times more stickers than Mary.

Mary → 1 unit *(not stickers)*

John → (1 + 5) units = 6 units

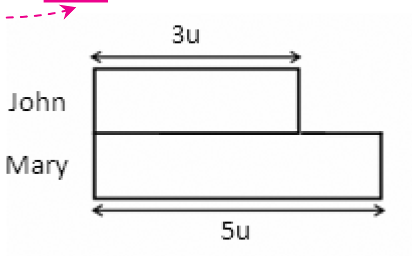


FRACTION

3. **John has $\frac{3}{5}$ as many stickers as Mary.**

That means, John has $\frac{3}{5}$ as many stickers as Mary.

Mary → 5 unit *(not stickers)*
John → 3 units

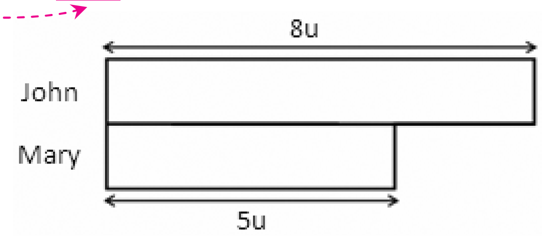


4. **John has $1\frac{3}{5}$ as many stickers as Mary.**

First, convert $1\frac{3}{5}$ to improper fraction. $1\frac{3}{5} = \frac{8}{5}$

That means, John has $\frac{8}{5}$ as many stickers as Mary.

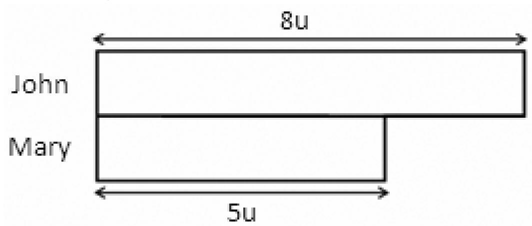
Mary → 5 unit *(not stickers)*
John → 8 units



5. **John has $\frac{3}{5}$ more stickers than Mary.**

That means, John has $\frac{3}{5}$ more stickers than Mary.
Or, John has 3 units *(not stickers)* more than Mary.

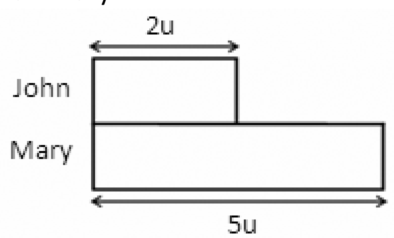
Mary → 5 units *(not stickers)*
John → (5 + 3) units = 8 units



6. **John has $\frac{3}{5}$ fewer stickers than Mary.**

That means, John has $\frac{3}{5}$ fewer stickers than Mary.
Or, John has 3 units *(not stickers)* fewer than Mary.

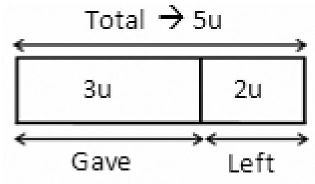
Mary → 5 units *(not stickers)*
John → (5 – 3) units = 2 units



7. John gave away $\frac{3}{5}$ of his stickers.

That means, John **gave** away $\frac{3}{5}$ of his **stickers**.

Total → 5 units *(not stickers)*
Gave → 3 units
Left → (5 – 3) units = 2 units



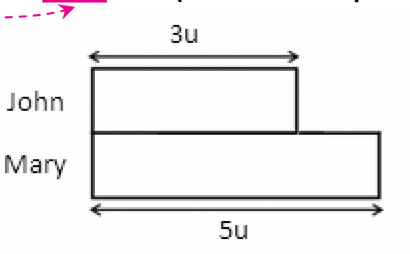
DECIMAL

8. John has 0.6 times <u>as many</u> stickers as Mary.

First, convert the decimal to fraction. $0.6 = \frac{6}{10} = \frac{3}{5}$

That means, **John** has $\frac{3}{5}$ **as many** stickers as **Mary**. **(Just like Example 3)**

Mary → 5 unit *(not stickers)*
John → 3 units

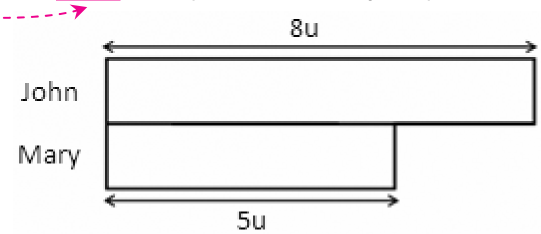


9. John has 1.6 times <u>as many</u> stickers as Mary.

First, convert the decimal to improper fraction. $1.6 = 1\frac{6}{10} = 1\frac{3}{5} = \frac{8}{5}$

That means, **John** has $\frac{8}{5}$ **as many** stickers as **Mary**. **(Just like Example 4)**

Mary → 5 unit *(not stickers)*
John → 8 units

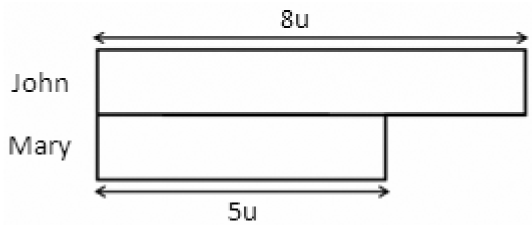


10. John has 0.6 times <u>more</u> stickers than Mary.

First, convert the decimal to fraction. $0.6 = \frac{6}{10} = \frac{3}{5}$

That means, **John** has $\frac{3}{5}$ **more** stickers than **Mary**. **(Just like Example 5)**
Or, John has 3 units *(not stickers)* more than Mary.

Mary → 5 units *(not stickers)*
John → (5 + 3) units = 8 units

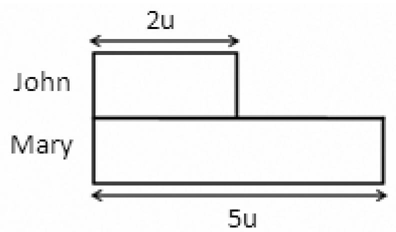


11. John has 0.6 times <u>fewer</u> stickers than Mary.

First, convert the decimal to fraction. $0.6 = \frac{6}{10} = \frac{3}{5}$

That means, **John** has $\frac{3}{5}$ **fewer** stickers than **Mary**. **(Just like Example 6)**
Or, John has 3 units *(not stickers)* fewer than Mary.

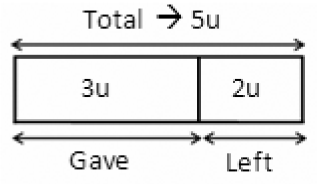
Mary → 5 units *(not stickers)*
John → (5 – 3) units = 2 units



12. John gave away 0.6 of his stickers.

First, convert the decimal to fraction. $0.6 = \frac{6}{10} = \frac{3}{5}$

That means, John **gave** away $\frac{3}{5}$ of his **stickers**. **(Just like Example 7)**



Total → 5 units *(not stickers)*
Gave → 3 units
Left → (5 – 3) units = 2 units

PERCENTAGE

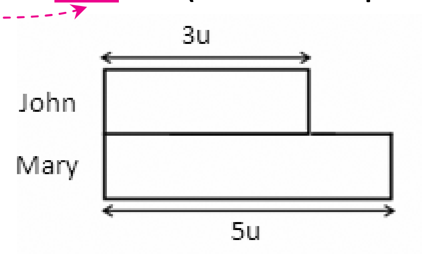
13. John has 60% as many stickers as Mary.

First, convert the percentage to fraction. 60% = $\frac{60}{100}$ = $\frac{3}{5}$

That means, **John** has $\frac{3}{5}$ **as many** stickers as **Mary**. **(Just like Example 3)**

Mary → 5 unit *(not stickers)*

John → 3 units



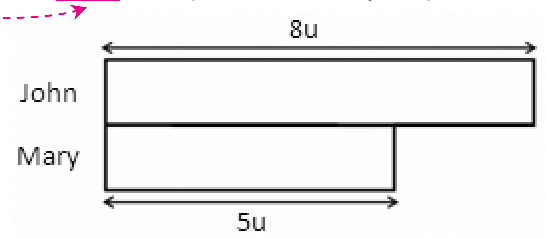
14. John has 160% as many stickers as Mary.

First, convert the percentage to improper fraction. 160% = $\frac{160}{100}$ = $\frac{8}{5}$

That means, **John** has $\frac{8}{5}$ **as many** stickers as **Mary**. **(Just like Example 4)**

Mary → 5 unit *(not stickers)*

John → 8 units



15. John has 60% more stickers than Mary.

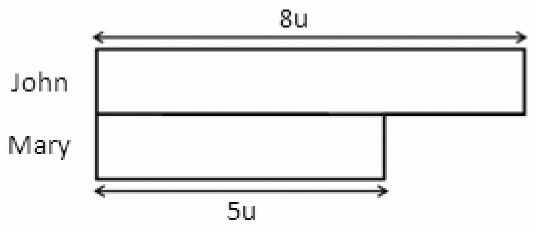
First, convert the percentage to fraction. 60% = $\frac{60}{100}$ = $\frac{3}{5}$

That means, **John** has $\frac{3}{5}$ **more** stickers than **Mary**. **(Just like Example 5)**

Or, John has 3 units *(not stickers)* more than Mary.

Mary → 5 units *(not stickers)*

John → (5 + 3) units = 8 units



16. John has 60% fewer stickers than Mary.

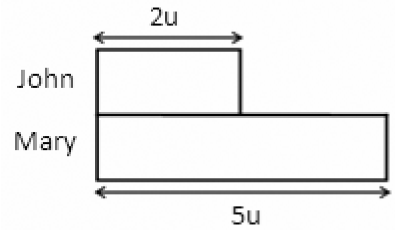
First, convert the percentage to fraction. 60% = $\frac{60}{100}$ = $\frac{3}{5}$

That means, **John** has $\frac{3}{5}$ **fewer** stickers than **Mary**. **(Just like Example 6)**

Or, John has 3 units *(not stickers)* fewer than Mary.

Mary → 5 units *(not stickers)*

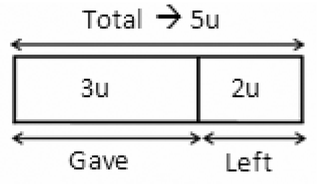
John → (5 – 3) units = 2 units



17. John <u>gave</u> away 60% of his stickers.

First, convert the decimal to fraction. 60% = $\frac{60}{100}$ = $\frac{3}{5}$

That means, John **<u>gave</u>** away $\frac{3}{5}$ of his **<u>stickers</u>**. **(Just like Example 7)**



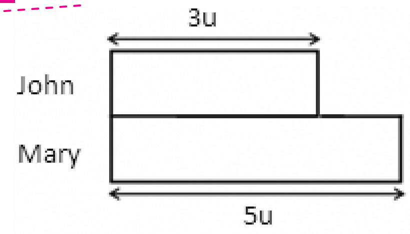
Total → 5 units *(not stickers)*
Gave → 3 units
Left → (5 – 3) units = 2 units

RATIO

18. John and Mary have stickers in the ratio of 3:5.

John and **Mary** have stickers in the **ratio** of 3:5.

Mary → 5 units *(not stickers)*
John → 3 units



PART 1

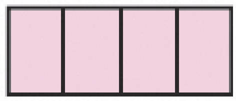
CONVENTIONAL MODEL

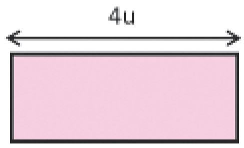
BEFORE YOU BEGIN FOR CONVENTIONAL MODEL

Conventional models may be illustrated in two ways – by drawing and by labelling.

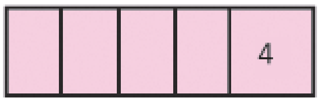
By drawing | **By labelling**

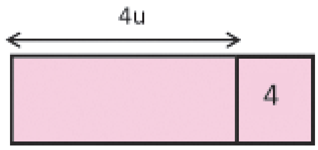
4 units



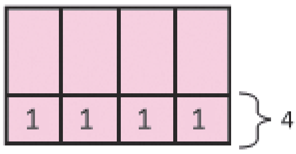


4 units + 4

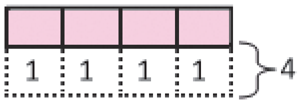


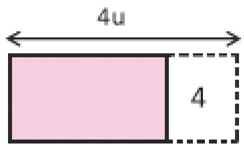


or



4 units – 4





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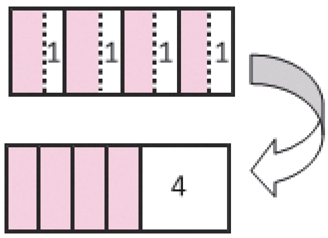


As a general rule:

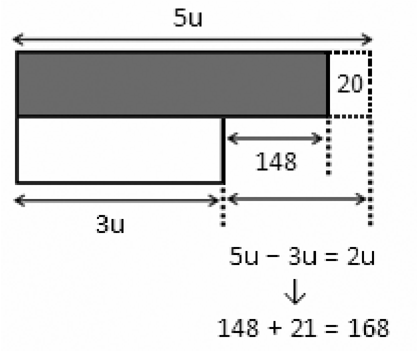
- use drawing when the quantity is less than 10 units
- use labelling when the quantity is 10 units or more, or when the quantity is negative (eg. subtraction).

CHAPTER 1 LABELLING THE BAR

EXAMPLES

1. **Substract 20 from 5 times the number A. The value will be 148 more than 3 times the number A. Find the value of number A.**

WORKING

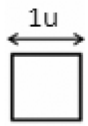


2 units → 168
1 unit → 168 ÷ 2 = 84

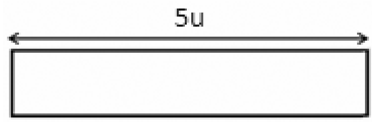
The value of number A is 84.

EXPLANATION

Number A is represented by

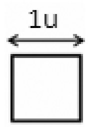


5 times the number A is thus

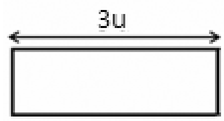


Subtract 20 from 5 times the number A

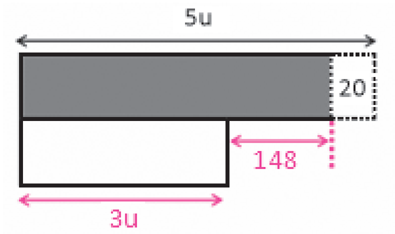
Number A is already represented by



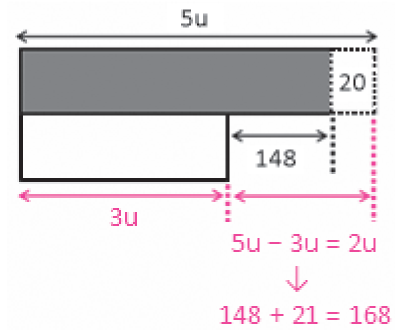
3 times the number A is thus



The value of "subtract 20 from 5 times the number A" is more than the value of "3 times the number A" by 148



Perform the calculation

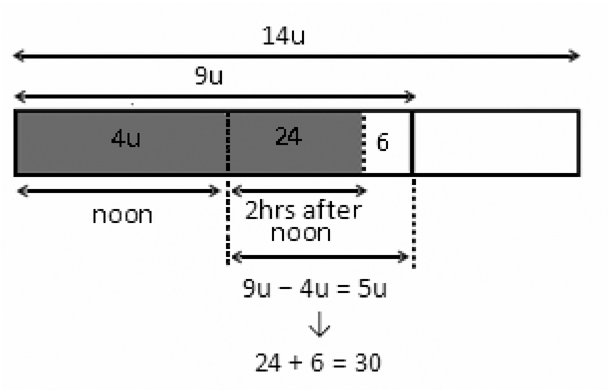


2 units → 168
1 unit → 168 ÷ 2 = 84

The value of number A is 84.

2. **There were some durians at a fruit stall. By noon, the number of durians sold was 40% of the number of durians left. Two hours later, another 24 durians were sold. The total number of durians sold was 6 less than $\frac{9}{14}$ of the number of durians in the fruit stall at first. How many durians were there at first?**

WORKING



5 units → 30
1 unit → 30 ÷ 5 = 6
14 units → 14 x 6 = 84

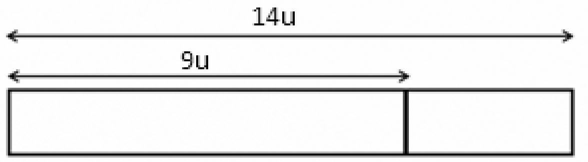
There were 84 durians at first.

EXPLANATION

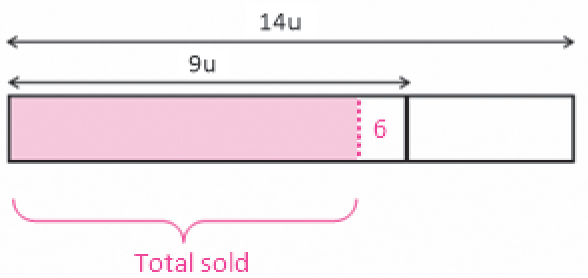
CONFUSION ALERT

The trick to solving this question is to work backwards, starting from the end of the question. The rationale is to always work from the total. From the phrase "$\frac{9}{14}$ of the number of durians in the fruit stall at first", the total can be represented as 14 units.

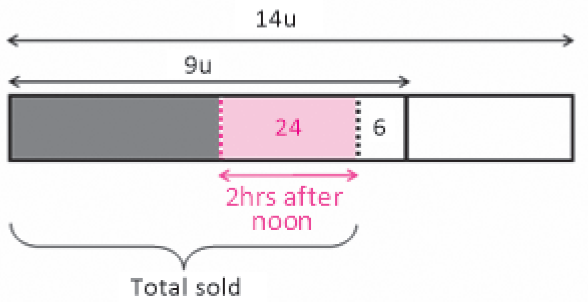
$\frac{9}{14}$ of the number of durians in the fruit stall at first is represented by



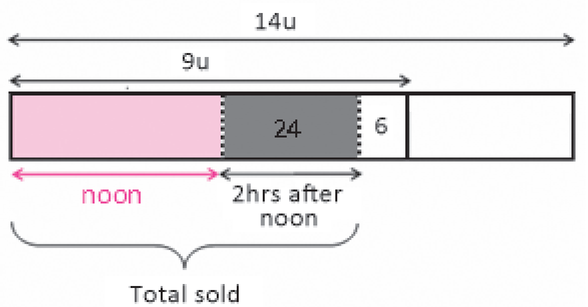
The total number of durians sold was 6 less than $\frac{9}{14}$



Out of the total number of durians sold, 24 durians were sold two hours after noon.



The remaining portion represents the number of durians sold by noon.



By noon, the number of durians sold was 40% of the number of durians left.

Convert the percentage to fraction. $40\% = \frac{40}{100} = \frac{2}{5}$

That means the number of durians sold is $\frac{2}{5}$ of the number of durians left.

Number of durians sold → 2 parts *(not number of durians)*
Number of durians left → 5 parts
Total number of durians at first → (2 + 5) parts = 7 parts

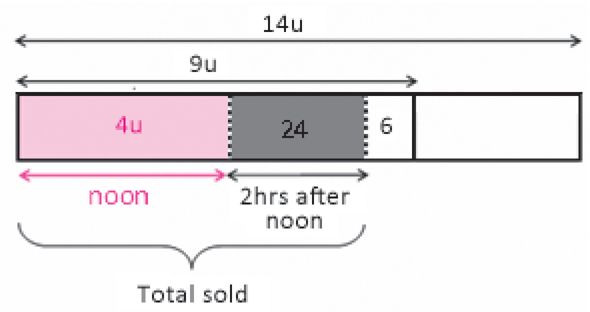
Sold	Left	Total
2 parts	5 parts	7 parts

CONFUSION ALERT

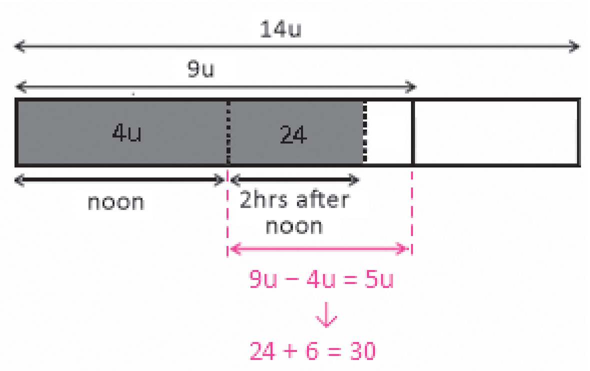
At whichever point in the day, the total number of durians in the fruit stall at first would remain constant. In the fraction $\frac{9}{14}$, the total number of durians in the fruit stall at first is represented as 14 units. In the fraction $\frac{2}{5}$, the total cannot be represented as 7 units because 14 units ≠ 7 units. Therefore, it is represented as 7 parts. This will later be multiplied by 2 to convert it to 14 units.

Earlier on, the total number of durians in the fruit stall at first was represented as 14 units. Hence, we must convert the 7 parts total to 14 units. We do this by multiplying the entire row by 2.

Sold	Left	Total
2 parts x 2 = 4 units	5 parts x 2 = 10 units	7 parts x 2 = 14 units



Perform the calculation



5 units → 30
1 unit → 30 ÷ 5 = 6
14 units → 14 x 6 = 84

There were 84 durians at first.

LET'S APPLY Problems Involving Labelling the Bar

1. Subtract 65 from 7 times the number A. The difference will be 295 more than 4 times the number. Find the value of number A.

2. Alex sold 55% of his oranges and threw away 20 rotten ones from the remaining oranges. The number of oranges sold was 40 more than the number of oranges left. How many oranges did he have at first?

3. There were some pears in a fruit stall. In the afternoon, the number of pears sold was 60% the number of pears left. In the evening, another 37 pears were sold. The total number of pears sold was 4 more than $\frac{9}{16}$ of the number of pears the fruit stall had at first. How many pears were there at first?

4. John bought some sweets to be shared equally among his 32 students. When 10 of the students gave up $\frac{4}{5}$ of their share, the rest of the students received 4 more sweets each. How many sweets did John buy?

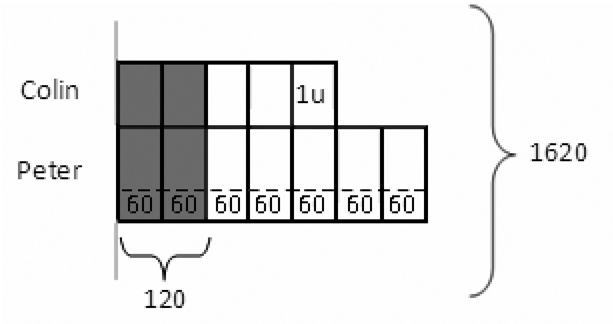
5. Adrian paid a total of $1025 for a chair, a dining table and a wardrobe. The wardrobe costs $296. The total cost of the chair and the dining table is $154 more than the total cost of the wardrobe and 3 such chairs. Find the cost of the chair.

CHAPTER 2 COMPARING THE BAR

EXAMPLES

1. **Colin and Peter have a total savings of $1620. 40% of Colin's savings is $120 less than $\frac{2}{7}$ of Peter's savings. Find the amount of money Colin saved.**

WORKING



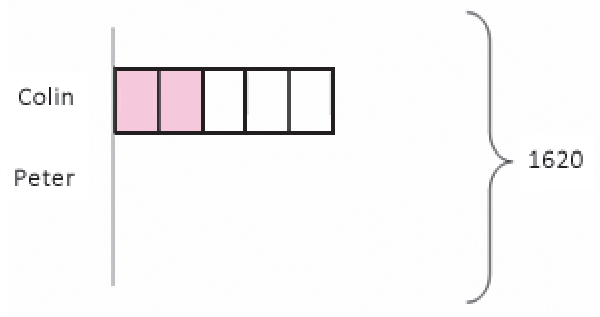
12 units → 1620 – 420 = 1200
1 unit → 1200 ÷ 12 = 100
5 units → 5 x 100 = 500

Colin saved $500.

EXPLANATION

Colin and Peter have a total savings of $1620.

Look at 40% of Colin's savings.
Convert the percentage to fraction.
40% = $\frac{40}{100}$ = $\frac{2}{5}$
That means we are looking at $\frac{2}{5}$ of Colin's savings.

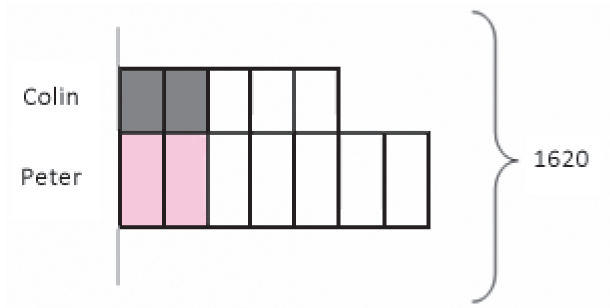


40% of Colin's savings is $120 less than $\frac{2}{7}$ of Peter's savings.

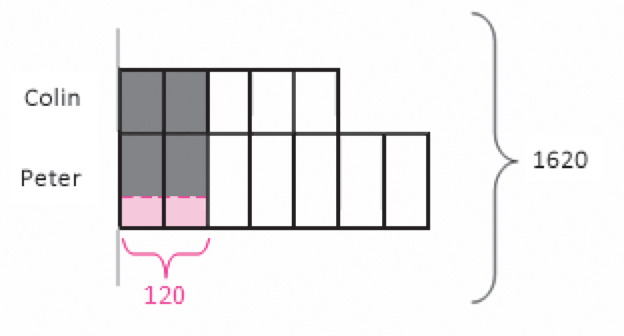
$\frac{2}{5}$ of Colin's savings is being compared with $\frac{2}{7}$ of Peter's savings.

$\frac{2}{7}$ of Peter's savings is more than $\frac{2}{5}$ of Colin's savings.

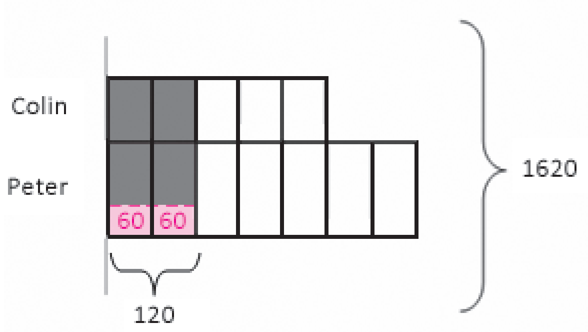
That means each part of Peter's savings is more than each unit of Colin's savings. In In the model, each portion of Peter's savings must be longer than each portion of Colin's savings.



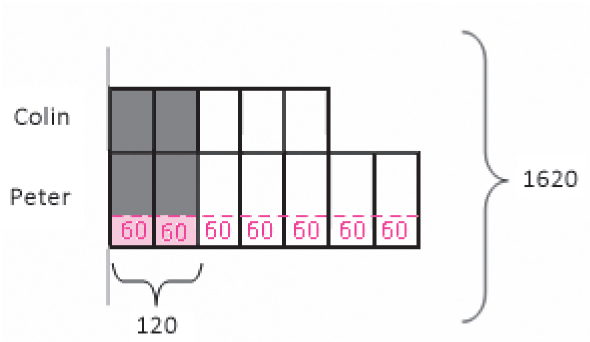
$\frac{2}{7}$ of Peter's savings is more than $\frac{2}{5}$ of Colin's savings by $120.



Each portion of Peter's savings is more than each potion of Colin's savings by ($120 ÷ 2) = $60.



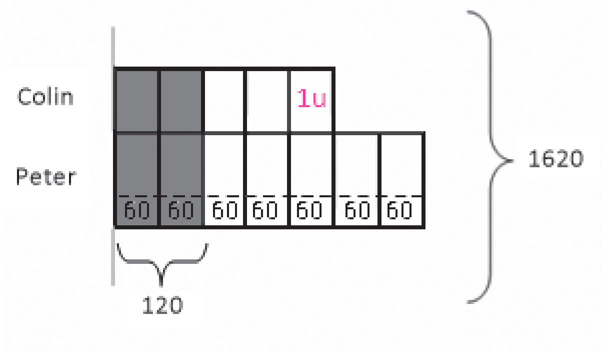
Extend the "more by $60" across all the portions of Peter's savings.



Now, perform the calculations.

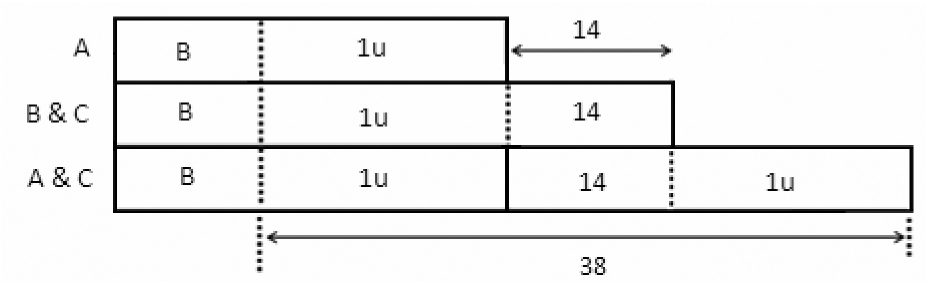
12 units → 1620 – (60 x 7) = 1200
1 unit → 1200 ÷ 12 = 100
5 units → 5 x 100 = 500

Colin saved $500.



2. Ali bought 3 packets of sweets. Packet B and Packet C have 14 more sweets than Packet A. Packet A and Packet C have 38 more sweets than Packet B. Given that Packet A has more sweets than Packet B, how many sweets are there in Packet C?

WORKING

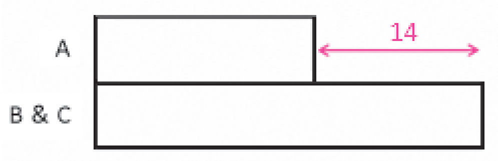


2 units	→	38 – 14 =	24
1 unit	→	24 ÷ 2 =	12
1 unit + 14	→	12 + 14 =	26

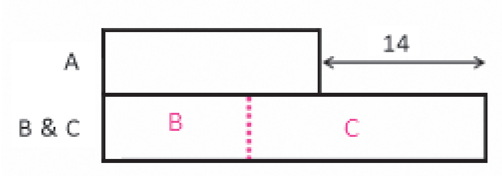
Packet C has 26 sweets.

EXPLANATION

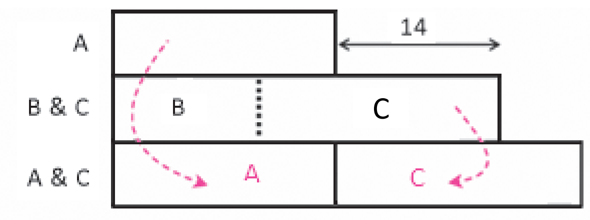
Packet B and Packet C have 14 more sweets than Packet A.



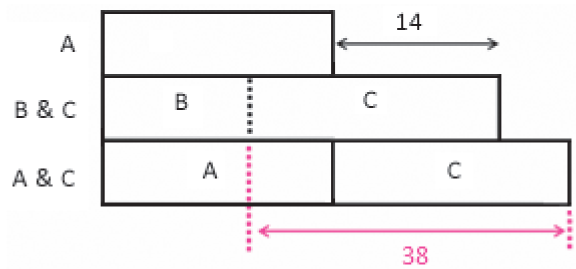
We know that Packet A has more sweets than Packet B.



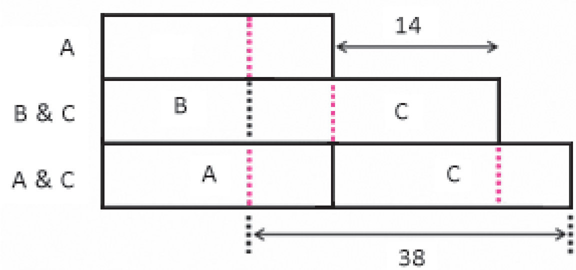
Packet A and Packet C have more sweets than Packet B.



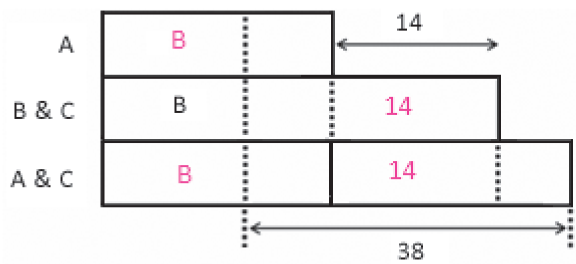
Packet A and Packet C have 38 more sweets than Packet B.



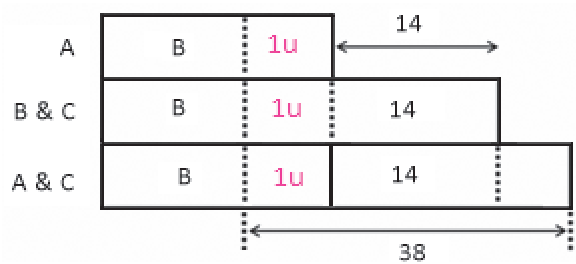
Extend all segmenting lines over the three bars.



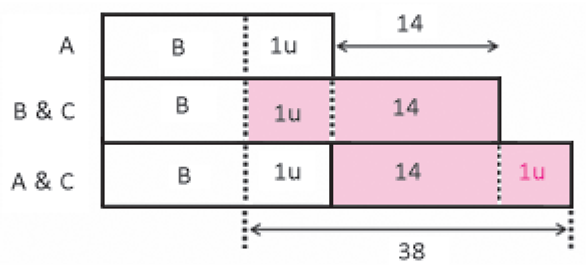
Label each of the more obvious portions (B and 14).



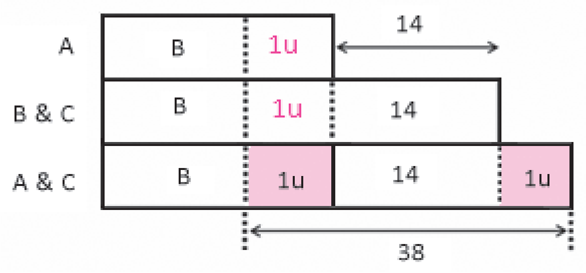
Define 1 unit.



In the 2nd bar, C = 1 unit + 14.
In the 3rd bar, we must also label such that C = 1 unit + 14.



Now, perform the calculations.

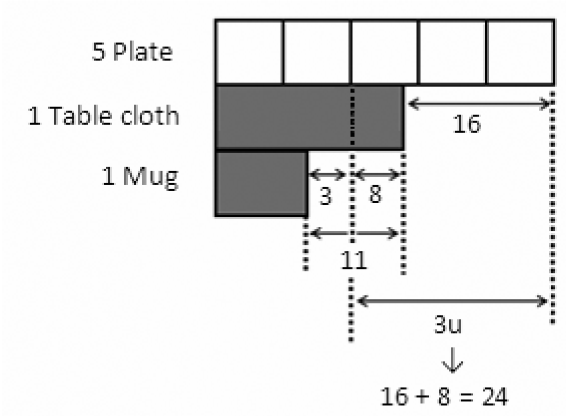


2 units	→	38 – 14 =	24
1 unit	→	24 ÷ 2 =	12
1 unit + 14	→	12 + 14 =	26

Packet C has 26 sweets.

3. **Brenda decided to buy a mug, a table cloth and a plate. The table cloth cost $16 less than 5 plates. The mug, which is $3 less than 2 plates, is $11 less than the table cloth. Find the cost of the plate.**

WORKING

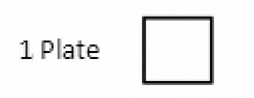


3 units → 24
1 unit → 24 ÷ 3 = 8

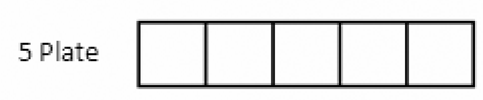
The cost of 1 plate is $8.

EXPLANATION

Cost of 1 plate is represented by



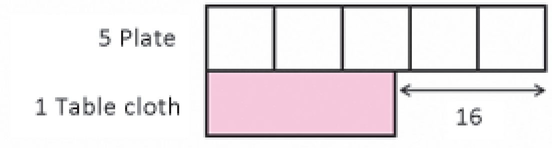
Cost of 5 plates is represented by



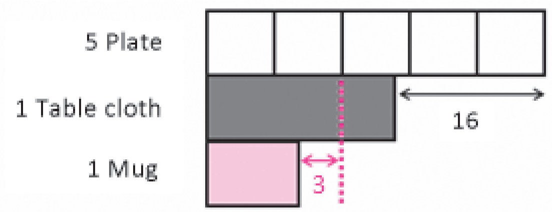
CONFUSION ALERT

We know that the bar for cost of 1 table cloth must be shorter than the bar for cost of 5 plates. However, we do not know how much shorter it should be. At this point we can only draw the bar for cost of 1 table cloth using a random length.

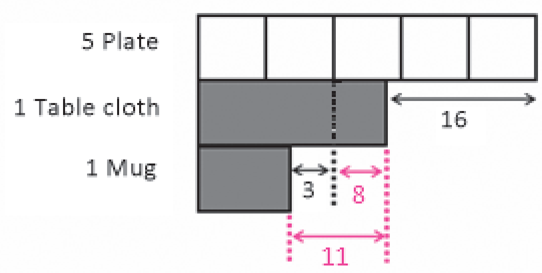
Cost of 1 table cloth is
less than cost of 5 plates
by $16



Cost of 1 mug is
less than cost of 2 plates
by $3



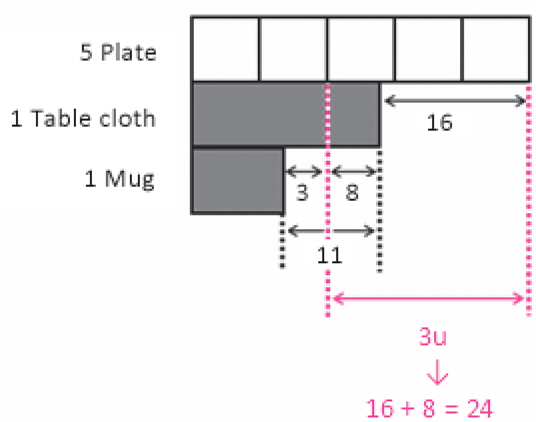
Cost of 1 mug is
less than cost of 1 table cloth
by $11



Now, perform the calculations.

3 units → 24
1 unit → 24 ÷ 3 = 8

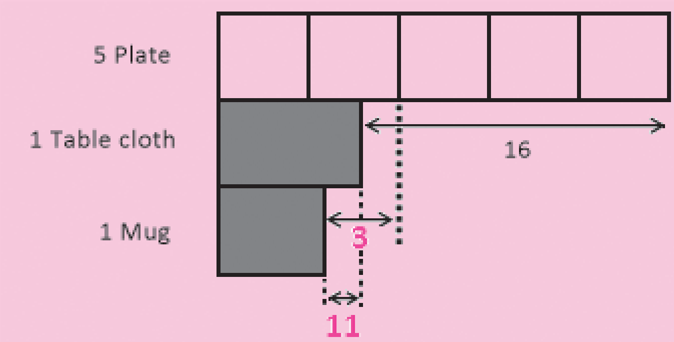
The cost of 1 plate is $8.



ALTERNATIVES

Earlier on, we drew the bar for cost of 1 table cloth using a random length.
What happens if we drew the bar much shorter or longer.

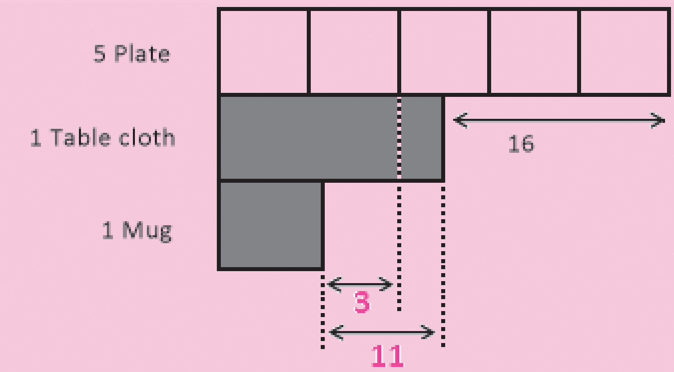
Drawing a much shorter bar



We end up with $11 being shorter than $3, which does not make sense.

That means, we cannot draw a much shorter bar.

Drawing a much longer bar



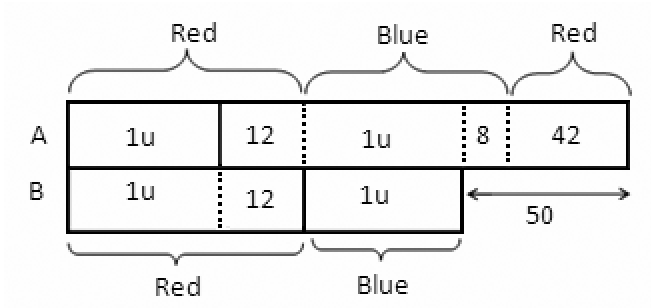
We end up with $11 being longer than $3, which makes sense.

That means, we can draw a much longer bar.

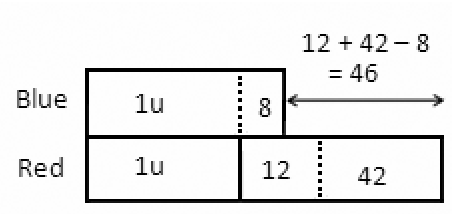
From the above variations and outcomes, it is obvious that the critical point is at the 2-plate length. The bar for cost of 1 table cloth must be longer than the 2-plate length.

4. **There are 50 more marbles in Box A than in Box B. In Box B, there are 12 more red marbles than blue marbles. There are 8 more blue marbles in Box A than in Box B. How many more red marbles than blue marbles are there in Box A?**

WORKING



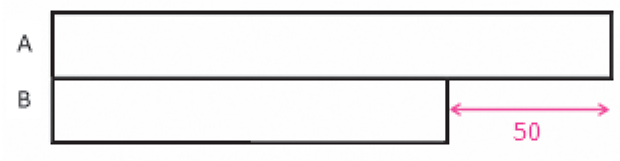
Regroup the marbles in Box A.



There are 46 more red marbles than blue marbles in Box A.

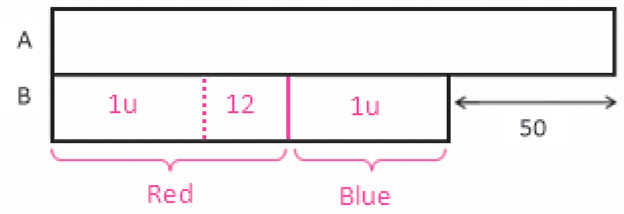
EXPLANATION

Box A has 50 more marbles than Box B.

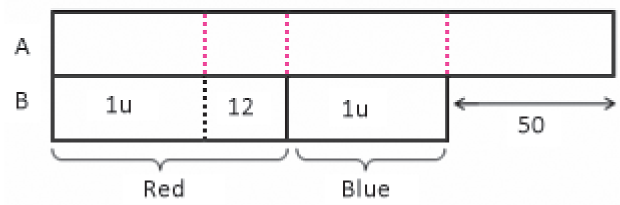


Box B has 12 red marbles more than blue marbles.

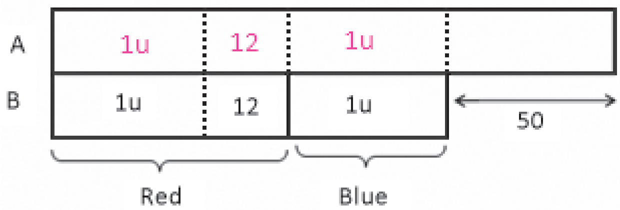
Assuming there is 1 unit of blue marbles, there is 1 unit + 12 red marbles.



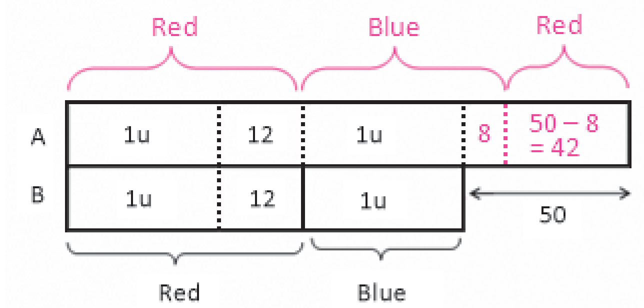
Extend all segmenting lines across the two bars.



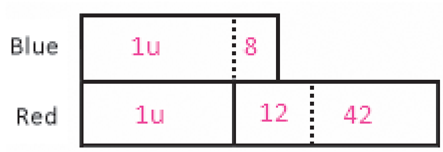
Label each of the more obvious portions (1 unit and 12).



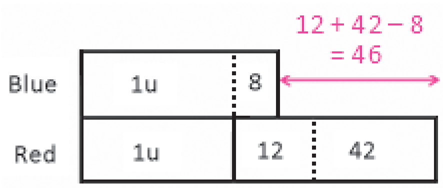
Box A has 8 more blue marbles than Box B.
That means, out of the 50 marbles in Box A, 8 are blue marbles.
The remaining 42 are red marbles.



In the model, regroup the marbles in Box A.



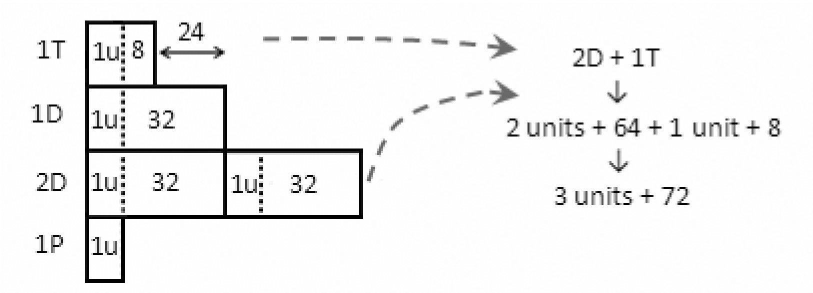
Now, perform the calculations.
There are 46 more red marbles than blue marbles in Box A.

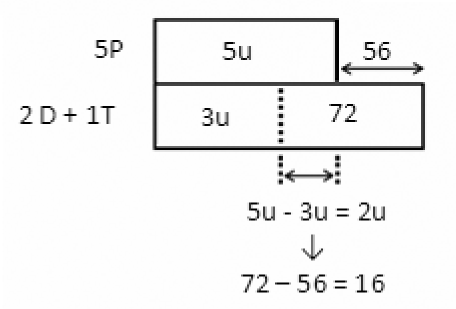


There are 46 more red marbles than blue marbles in Box A.

5. **Mrs Tan bought a T-shirt, 2 similar dresses and 4 similar purses. The total cost of the dresses and T-shirt is $56 more than the total cost of the 5 purses. The T-shirt cost $24 less than each dress. Each dress cost $32 more than each purse. Find the cost of each dress.**

WORKING





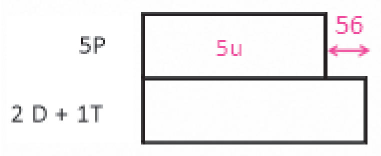
2D + 1T → 2 units + 64 + 1 unit + 8
→ 3 units + 72

2 units → 16
1 unit → 16 ÷ 2 = 8
1 unit + 32 → 8 + 32 = 40

Each dress cost $40.

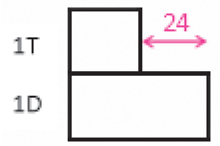
EXPLANATION

The dresses and T-shirt cost more than the 5 purses by $56

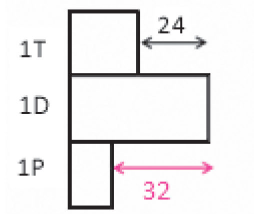


Start a second model.

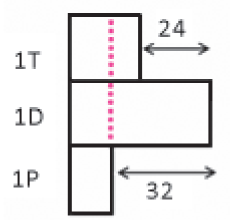
The T-shirt cost less than each dress by $24



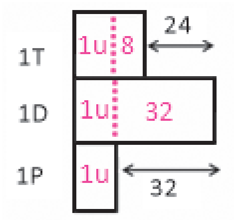
Each dress cost more than each purse by $32



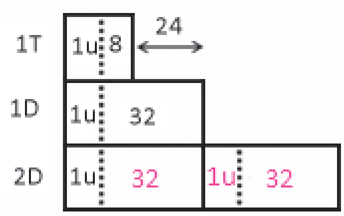
Extend the segmenting line of the purse across the two bars above it.



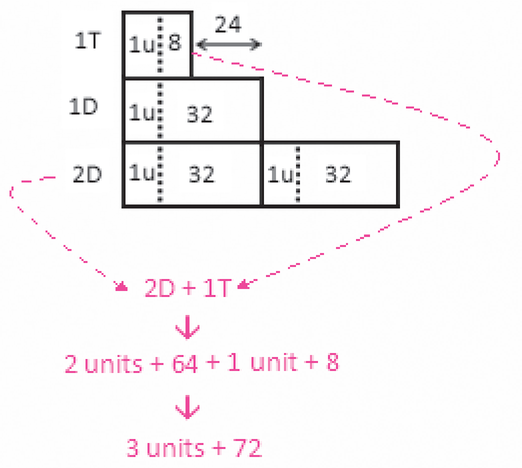
Label all the obvious portions (1 unit, 8 and 32).
5P → 5 units
1P → 5 ÷ 5 unit = 1 unit
So, the two similar areas above 1P is also 1 unit.



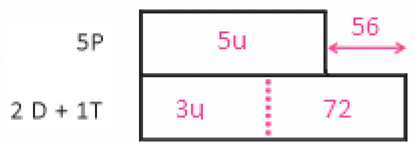
From the first bar, 1T → 1 unit + 8
From the second bar,
double everything to get 2D = 2 units + 64



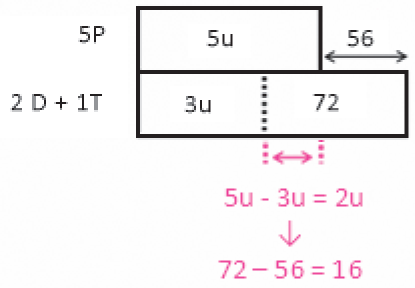
Add the first and third bar.
2D + 1T → 2 units + 64 + 1 unit + 8
→ 3 units + 72



Re-label the first model's second bar, which is 2D + 1T, to 3 units + 72



Now, perform the calculations.

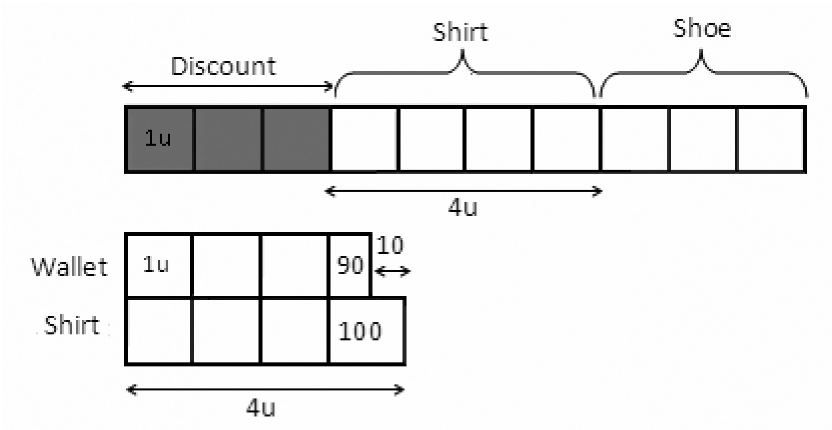


2 units → 16
1 unit → 16 ÷ 2 = 8
1 unit + 32 → 8 + 32 = 40

Each dress cost $40.

6. **At a year-end sale, Gene bought a shirt and a pair of shoes. With the saving from the 30% discount given to the two items and another $90, he bought a wallet. The shirt cost $10 more than the wallet. Given that the price of the shirt was $1\frac{1}{3}$ of the price of the pair of shoes, how much did Gene pay for all the items?**

WORKING



Now we know:	1 unit	→	100		
Cost of shirt:	4 units	→	4 x 100	=	400
Cost of wallet:	3 units + 90	→	(3 x 100) + 90	=	390
Cost of shoes:	3 units	→	3 x 100	=	300
Add all the costs:	400 + 390 + 300 = 1090				

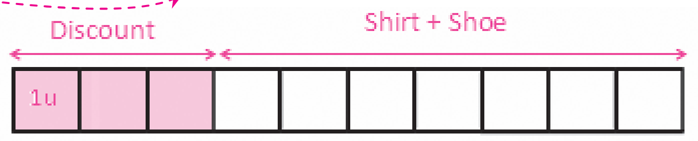
Gene paid $1090 for all the items.

EXPLANATION

Gene had a 30% discount on the two items he bought.

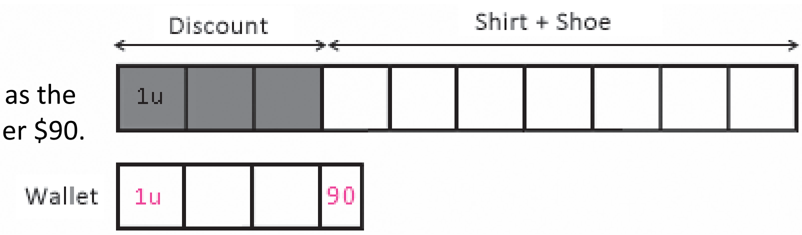
Convert the percentage to fraction. $30\% = \frac{30}{100} = \frac{3}{10}$

That means, Gene had a $\frac{3}{10}$ discount on the two items he bought.

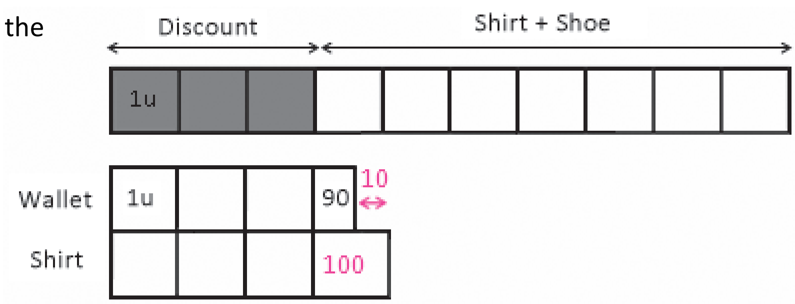


Start a second model.

The wallet cost as much as the 30% discount and another $90.



The shirt cost more than the wallet by $10



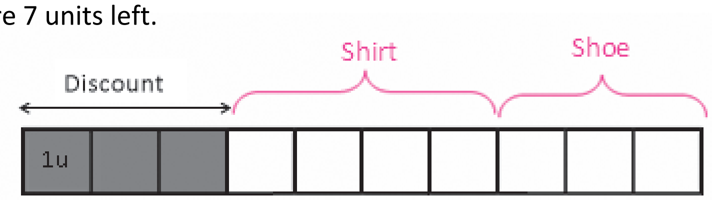
The shirt cost $1\frac{1}{3}$ times the pair of shoes

Convert $1\frac{1}{3}$ to improper fraction. $1\frac{1}{3} = \frac{4}{3}$

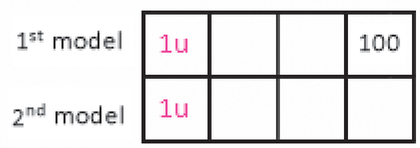
That means, the shirt cost $\frac{4}{3}$ compared to the shoes.

Shirt → 4 unit *(shirts)*
Shoes → 3 units
Total → 4 + 3 units = 7 units

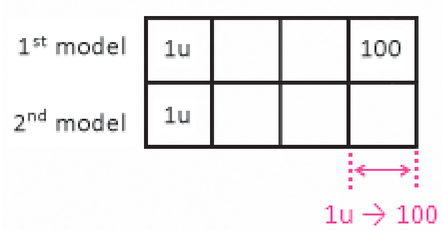
Label the first model's bar accordingly.
After the 30% discount, there are 7 units left.
4 units represent cost of shirt.
3 units represent cost of shoes.



Compare first model's and second model's price of shirt.



Now, perform the calculations.



Now we know:	1 unit	→	100		
Cost of shirt:	4 units	→	4 x 100	=	400
Cost of wallet:	3 units + 90	→	(3 x 100) + 90	=	390
Cost of shoes:	3 units	→	3 x 100	=	300
Add all the costs:	400 + 390 + 300 = 1090				

Gene paid $1090 for all the items.

LET'S APPLY Problems Involving Comparing the Bar

1. Brandon decided to buy a calculator, a pencil box and a book. The cost of the calculator is $26.10 less than 8 times the cost of the book. The cost of the pencil box, which is $10.30 less than 3 times the cost of the book, is $14.20 less than the cost of the calculator. Find the cost of the book.

2. Keith and Philip have a total of 298 stamps. $\frac{1}{5}$ of Keith's stamps is 90 fewer than $\frac{3}{7}$ of Philip's stamps. Find the number of stamps each of them has.

3. Amy bought some stationery from a bookstore. The total cost of 3 files, File A, File B and File C, was $5.60 more than the total cost of 4 staplers. File A costs $1.90 less than each of the other 2 files. File B costs $4.20 more than each of the staplers. Find the cost of File C.

4. Benny has $\frac{1}{3}$ as many stamps as Ali. Benny has 100 stamps less than Charles. Charles has 20 stamps more than Ali. Find the total number of stamps that Ali, Benny and Charles have.

5. Freddy is thinking of four numbers. The first number is the same as the average of the four numbers. The second number is 2 less than the average of the four numbers. The third number is 9 more than the average of the four numbers. Given that the fourth number is 36, find the sum of the four numbers.

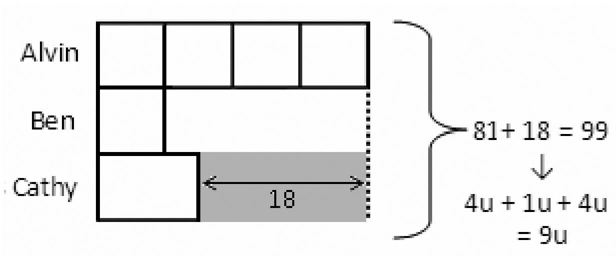
6. There are 90 more pupils in Tuition Centre A than in Tuition Centre B. In Tuition Centre B, there are 36 more girls than boys. There are 24 more boys in Tuition Centre A than in Tuition Centre B. How many more girls than boys are there in Tuition Centre A?

CHAPTER 3 COMPLETING THE BAR

EXAMPLES

1. **Alvin is 4 times as old as Ben. Cathy is 18 years younger than Alvin. Their total age is 81. How old is Cathy?**

WORKING



9 units → 99
1 unit → 99 ÷ 9 = 11

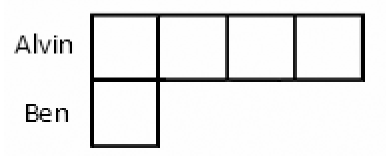
Cathy's age is 4 units – 18.

4 units – 18 → (4 x 11) – 18 = 26

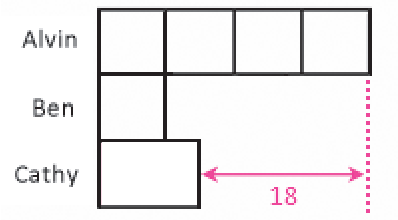
Cathy is 26 years old.

EXPLANATION

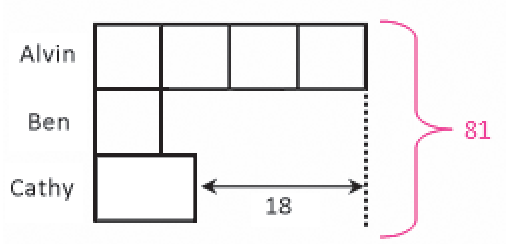
Alvin is 4 times as old as Ben.



Cathy is younger than Alvin by 18 years.

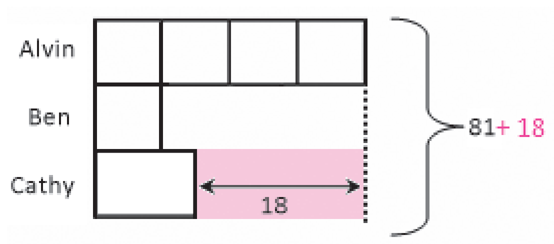


Their total age is 81.

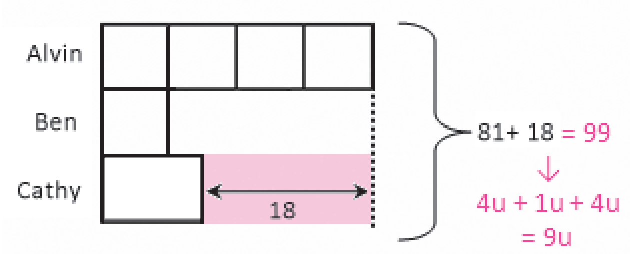


Make Cathy's bar equal to Alvin's bar by adding 18 years to Cathy's bar.

This means their total age is increased by 18 years.



Now, perform the calculations.



9 units → 99
1 unit → 99 ÷ 9 = 11

Cathy's age is 4 units – 18.

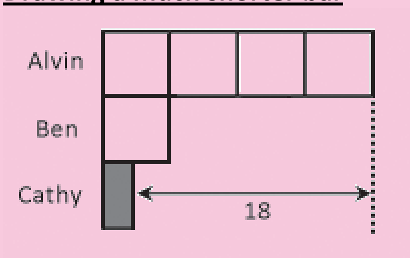
4 units – 18 → (4 x 11) – 18 = 26

Cathy is 26 years old.

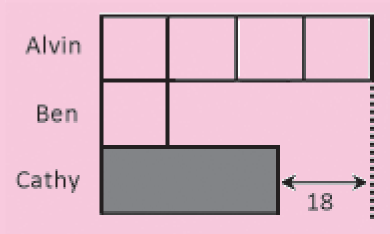
ALTERNATIVES

Earlier, we drew the bar for Cathy's age using a random length.
What happens if we draw the bar much shorter or longer.

<u>Drawing a much shorter bar</u>

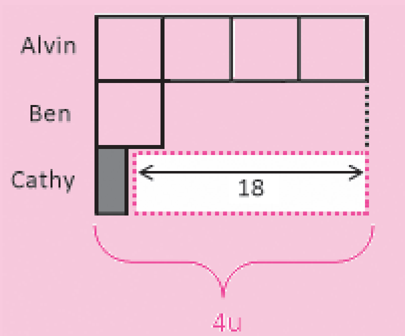


<u>Drawing a much longer bar</u>

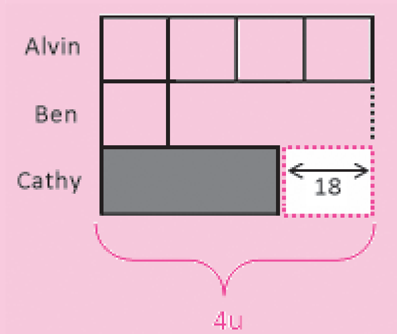


Make Cathy's bar equal to Alvin's bar
by adding 18 years to Cathy's bar.

This means their total age is increased
by 18 years.



9 units → 99



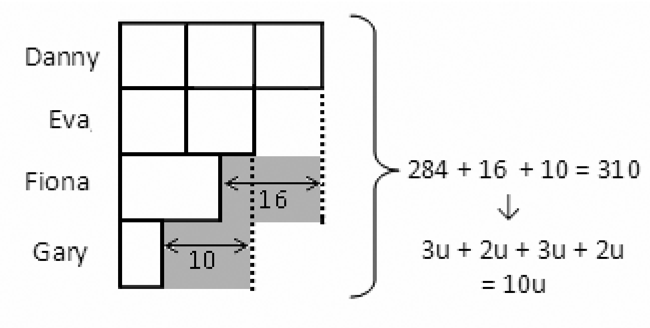
9 units → 99

Both variations result in the discovery that 9 units → 99.

For this particular question, the solution process involves making one length (Cathy's bar) equal to another of known number of units (Alvin's bar of 4 units).

2. **In a mathematics test, Danny scored 1.5 times as much as Eva, and 16 marks more than Fiona. Gary scored 10 marks less than Eva. Their total score was 284. What was Fiona's score?**

WORKING



10 units → 310
1 unit → 310 ÷ 10 = 31

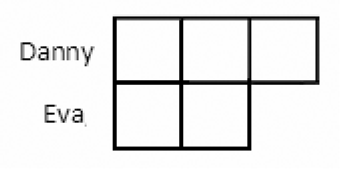
Fiona's score was 3 units – 16.

3 units – 16 → (3 x 31) – 16 = 77

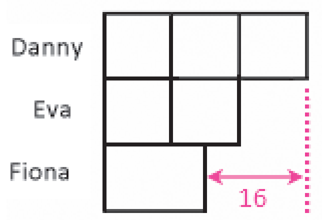
Fiona scored 77 marks.

EXPLANATION

Danny scored 1.5 times as much as Eva.
Convert the decimal to fraction. $1.5 = \frac{3}{2}$
That means, Danny scored $\frac{3}{2}$ times as much as Eva.
Danny → 3 units *(not score)*
Eva → 2 units



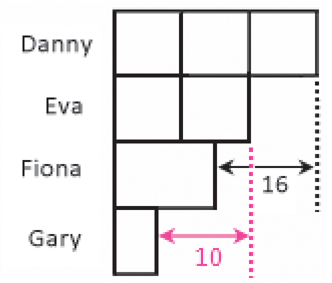
Danny scored more than Fiona by 16 marks.



CONFUSION ALERT

For this particular question, the solution process eventually involves making one length (Fiona's bar) equal to another of known number of units (Danny's bar of 3 units).

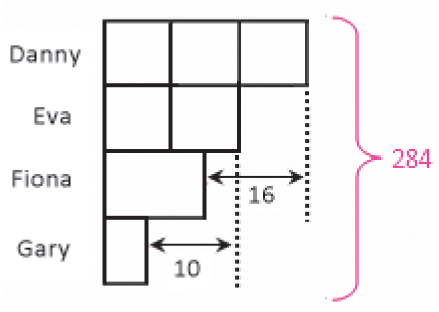
Gary scored less than Eva by 10 marks.



CONFUSION ALERT

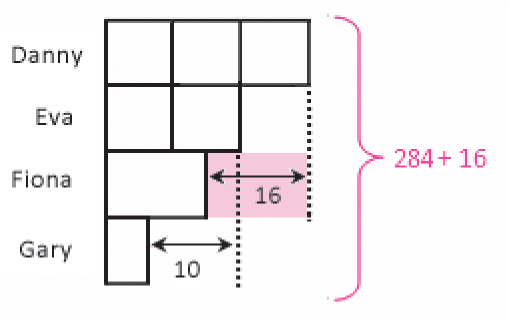
Again, for this particular question, the solution process eventually involves making one length (Gary's bar) equal to another of known number of units (Eva's bar of 2 units).

Their total score was 284.



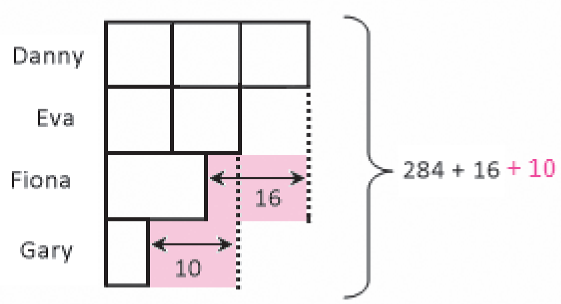
Make Fiona's bar equal to Danny's bar by adding 16 marks to Fiona's bar.

This means their total age is increased by 16 marks.

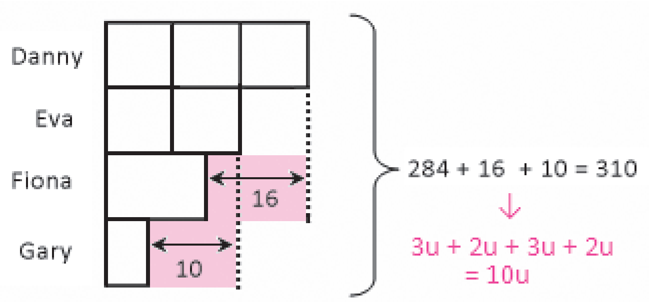


And make Gary's bar equal to Eva's bar by adding 10 marks to Gary's bar.

This means their total age is increased by 10 marks.



Now, perform the calculations.



10 units → 310
1 unit → 310 ÷ 10 = 31

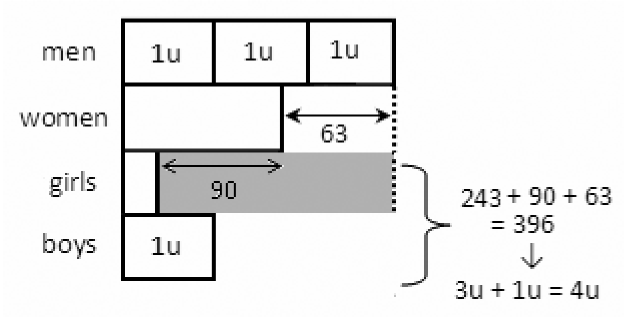
Fiona's score was 3 units – 16.

3 units – 16 → (3 x 31) – 16 = 77

Fiona scored 77 marks.

3. **At a concert, there were 63 more men than women. There were 90 fewer girls than women. The number of boys was $\frac{1}{3}$ that of the men. Given that the number of children was 243, find the total number of people at the concert.**

WORKING



4 units → 396
1 unit → 396 ÷ 4 = 99

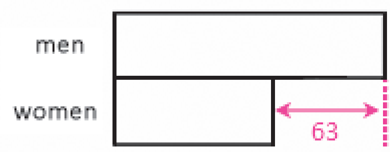
Men: 3 units = (3 x 99) = 297
Women: 3 units – 63 = (297 – 63) = 234
Children: 243

All: 297 + 234 + 243 = 774

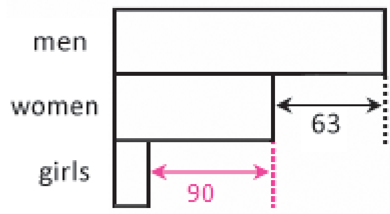
The total number of people at the concert is 774.

EXPLANATION

There were more men than women by 63.

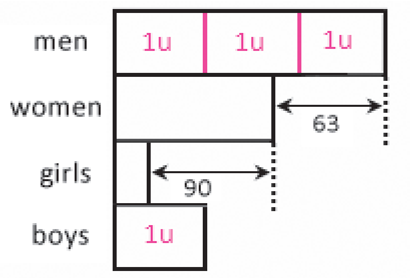


There were fewer girls than women by 90.

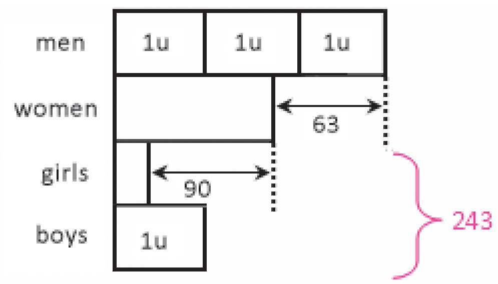


Number of boys were $\frac{1}{3}$ the number of men.

Boys → 1 unit *(not individuals)*
Men → 3 units

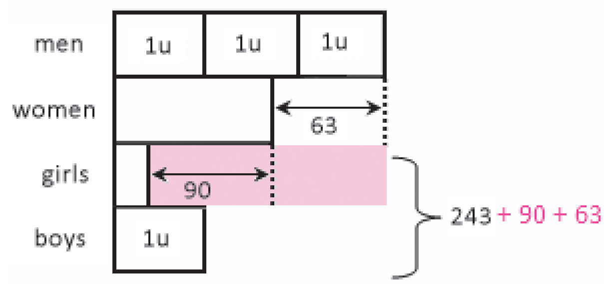


The number of children was 243.



Make girls' bar equal to men's bar by adding (90 + 63) marks to girls' bar. This makes the girls' bar 3 units.

This means the number of children is increased by (90 + 63) marks.

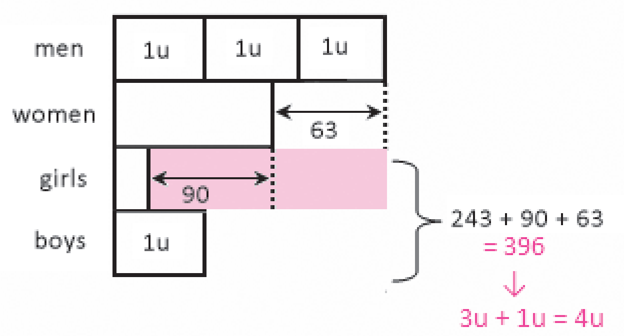


Now, perform the calculations.

4 units → 396
1 unit → 396 ÷ 4 = 99

Men: 3 units = (3 x 99) = 297
Women: 3 units – 63 = (297 – 63) = 234
Children: 243 *(already given)*

All: 297 + 234 + 243 = 774



The total number of people at the concert is 774.

LET'S APPLY Problems Involving Completing the Bar

1. Lina has 4 times as much money as Mary and $240 more than Norah. Given that Mary and Norah have $660, find how much Norah has.

2. Ali had 4 times as much money as Benny. Charlie had $60 less than Ali, and $40 more than Daniel. Given that the 4 children have a total of $490, find the amount of money Charlie and Daniel had altogether.

3. Patsy, Steven and Colin won $1100 cash from a lucky draw. Steven received $100 less than thrice the amount received by Patsy. Colin received $100 less than Steven. Find the amount of money received by Steven.

4. Minghua spent $12 on 4 similar pens, a ruler and a file. The ruler cost $1.20. The total cost of the 4 pens and the ruler was $2.40 more than the cost of the file. Find the cost of each pen.

5. The total cost of 25 foldable chairs, a metal cabinet and a writing desk is $1200. The metal cabinet costs $230. The total cost of the 25 foldable chairs is $650 more than the cost of the writing desk. Find the cost of each foldable chair.

6. A survey was conducted to find out consumers' preference to Drink A, Drink B, Drink C and Drink D. $\frac{5}{8}$ as many people voted for Drink D as for Drink A. 50 fewer people voted for Drink B than for Drink A. 26 more people voted for Drink B than for Drink C. Given that a total of 80 people voted for Drink C and Drink D, find the total number of people who participated in the survey.

ANSWERS

Answers to questions in the prior chapters' Let's Apply sections are listed in this chapter. Detailed workings may be downloaded at:

www.mathsheuristics.com/?page_id=472

PART 1 CONVENTIONAL MODEL

CHAPTER 1 LABELLING THE BAR

1. 120
2. 200 oranges
3. 176 pears
4. 352 sweets
5. $93

CHAPTER 2 COMPARING THE BAR

1. $6
2. Keith has 60 stamps
   Philip has 238 stamps
3. $9.30
4. 300 stamps
5. 172
6. 78 more girls than boys

CHAPTER 3 COMPLETING THE BAR

1. $480
2. $240
3. $500
4. $1.50
5. $32.40
6. 222 people

Mastering Heuristics Series
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Book

Heuristics is a specialised problem-solving concept now taught at primary-level maths. Merged into the regular maths curriculum, it is difficult to isolate and learn, making maths confusing to some. Many parents must set aside the regular-syllabus they learned in their youth to re-learn and teach their children heuristics – much easier said than done!

To give parents and students a complete and comprehensive guide to heuristics, Sunny Tan, Principle Trainer of mathsHeuristics™, wrote this series of books. The series neatly packages heuristics by techniques (series of books) and various well-defined application scenarios (chapters in each book). It also offers many examples, showing the efficiency of – and step-by-step application of – heuristics techniques.

Each book gives students the opportunity to see how a specific heuristics technique works, and to get in some practice. For students enrolled in mathsHeuristics™ programmes, it serves as a study companion, while keeping parents well-informed of what their children are learning.

This particular book specifically teaches the Model Approach to Problem-solving – the use of drawings to effectively analyse and solve challenging maths problems. This highly-visual problem-solving technique complements the Unit Transfer Method, and paves the way for students to eventually learn the algebraic approach in secondary school. Most importantly, it is a heuristics technique that is most commonly emphasised in school.

Author

Sunny Tan currently trains students in the application of various heuristics concepts, with special focus on students in their critical year – the PSLE year. He also conducts heuristics workshops for parents and educators.

For over 10 years in the 90s, NIE-trained Sunny taught primary and secondary maths in various streams. He observed how the transformed primary maths syllabus stumped children, parents and, sometimes, even teachers. How do you teach young children to accurately choose and sequentially apply different situational logic in solving non-routine problems? Sunny resolved to simplify the learning and application of such skills. Through years of research and development, Sunny eventually established the mathsHeuristics™ programme. Result-oriented research has since proven the consistent effectiveness of the mathsHeuristics™ programme.

Sunny's ingenious methodology has attracted much media interest – The Straits Times, The Business Times, TODAY, FM93.8 and various leading parenting magazines – as well as raving reviews by academia and parents.

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