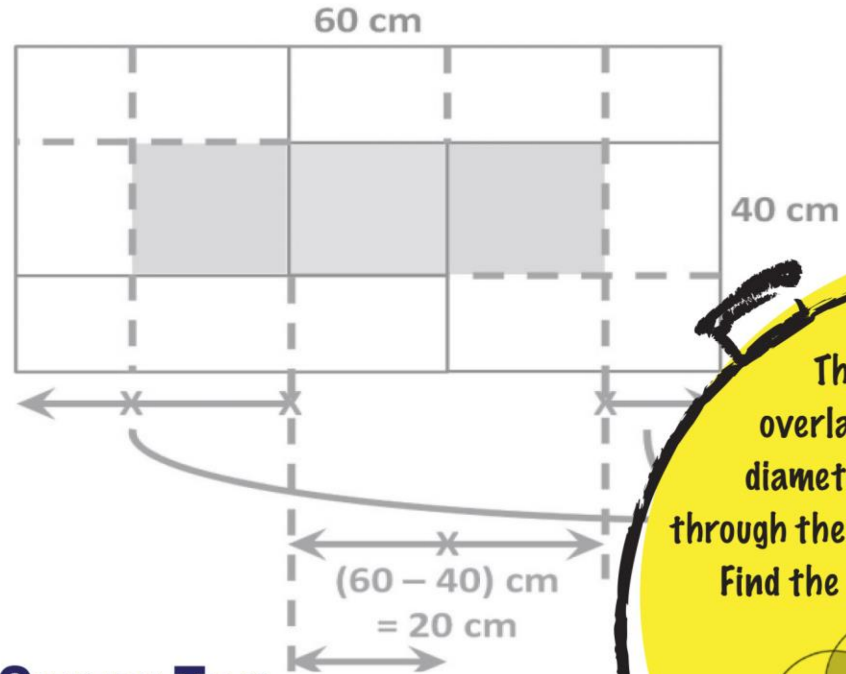
Mastering Heuristics Series
Handbooks for discerning parents

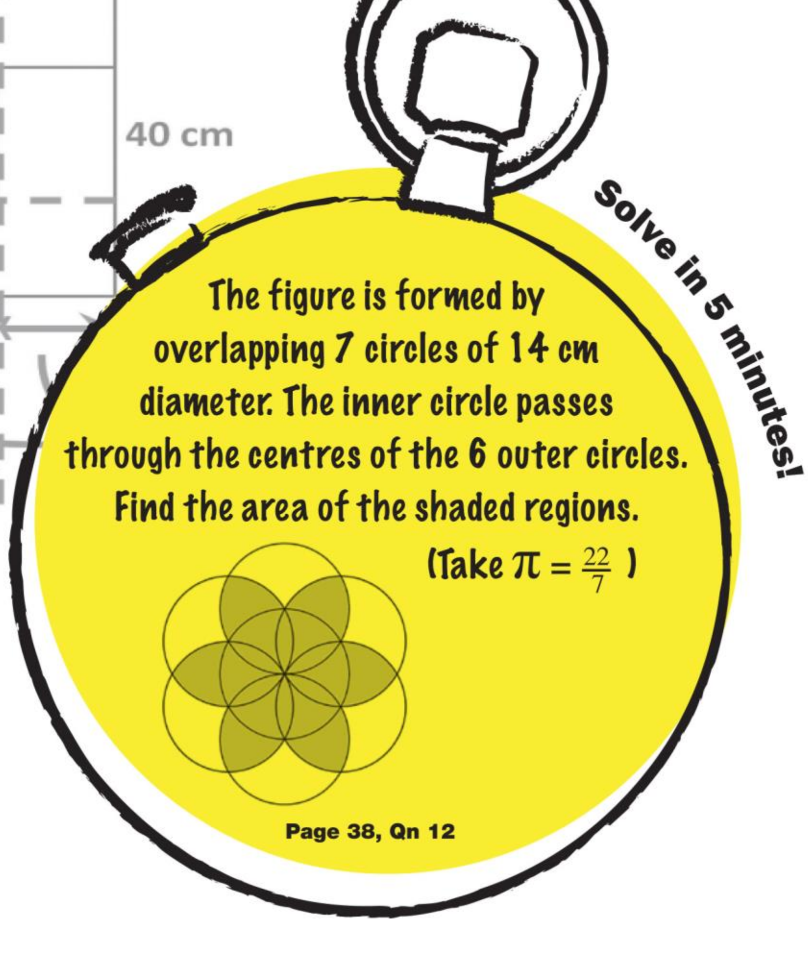
Spatial Visualisation

A Problem-solving Tool

for Challenging Problems in Area and Perimeter (Primary 5 & 6)



Sunny Tan



Mastering Heuristics Series
Handbook for discerning parents

Spatial Visualisation
A Problem-solving Tool
for Challenging Problems
in Area and Perimeter
(Primary 5 & 6)

Sunny Tan
Maths Heuristics Private Limited

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CONTENTS

* **Chapter 2**	Completely involves Circles Scenarios, hence entire chapter is for Primary 6 only.
** **Chapters 3, 5, 6 and 7**	Problems in Examples and Let’s Apply sections are grouped into Non-Circles and Circles Scenarios.

PREFACE

Heuristics in Primary Maths Syllabus

Heuristics is a specialised mathematical problem-solving concept. Mastering it facilitates efficiency in solving regular as well as challenging mathematical problems. The Ministry of Education in Singapore has incorporated 11 Problem-solving Heuristics into all primary-level mathematics syllabuses.

Learning Heuristics Effectively

However, the 11 Problem-solving Heuristics are not taught systematically; they have been dispersed into the regular curriculum. This not only makes it difficult for students to pick up Heuristics skills, but can also make mathematics confusing for them. For us parents, it is difficult to put aside the regular-syllabus mathematical concepts we were brought up on to re-learn Heuristics, much less teach our own children this new concept.

Take the Heuristic technique of Algebraic Equations, for instance. Parents and educators may attempt to teach their children to solve complex mathematical questions using Algebraic Equations. This will only confuse their children as many are too young to grasp the abstract concept. Instead, other Heuristics techniques should be used, according to current primary-level mathematics syllabus.

These and other challenges were what I observed firsthand during my years as a mathematics teacher, and provided me the impetus for my post-graduate studies, mathsHeuristics™ programmes and the Mastering Heuristics Series of guidebooks.

About Mastering Heuristics Series

This series of books is a culmination of my systematic thinking and experience, supported by professional instructional writing and editing, to facilitate understanding and mastery of Heuristics. I have neatly packaged Heuristics into main techniques (series of guidebooks) and mathematical scenarios (chapters within each guidebook). For each mathematical scenario, I offer several examples, showing how a particular heuristics technique may be applied, and then explaining the application in easy-to-follow steps and illustrations – without skipping a beat.

This particular guidebook in the series deals specifically with *Spatial Visualisation Techniques* for *Area and Perimeter* – the use of the mind's eye to manipulate given shapes to solve problems in Area and Perimeter. Mastery of this technique is necessary for solving especially-complex problems involving composite shapes. Spatial Visualisation is a very powerful problem-solving technique because it helps students literally see the solution to the problem posed.

The Mastering Heuristics Series provides a comprehensive guide to Heuristics. While each guidebook introduces parents to how Heuristics works, students have the opportunity to see the technique applied in different scenarios and to get in some practice. For students enrolled in mathsHeuristics™ programmes, each guidebook serves as a great companion, while keeping parents well-informed of what their children are learning.

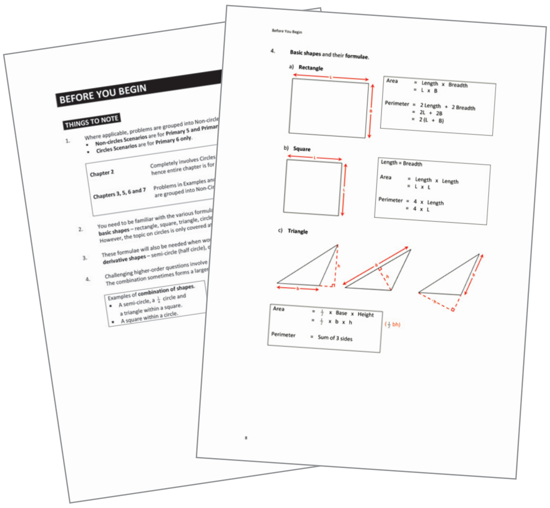
Sunny Tan
February 2014

HOW TO USE THIS BOOK

BEFORE YOU BEGIN

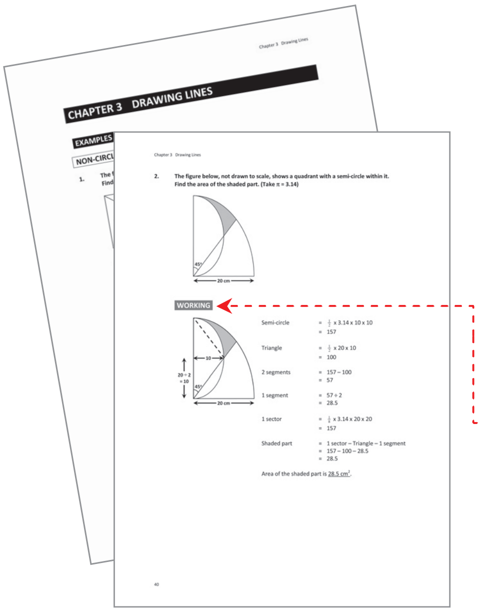
This chapter instills basic but important steps and truths in the heuristics technique that must be applied across every question in the guidebook. This helps to standardise the given information for easy application of the technique being taught.

In this guidebook on Spatial Visualisation, the steps include being familiar with parts and formulae of basic and derivative shapes.



CHAPTERS AND SECTIONS

Various scenarios are neatly separated into different chapters and sections. This allows the heuristics technique to be learnt and applied in a focused manner.



EXAMPLES

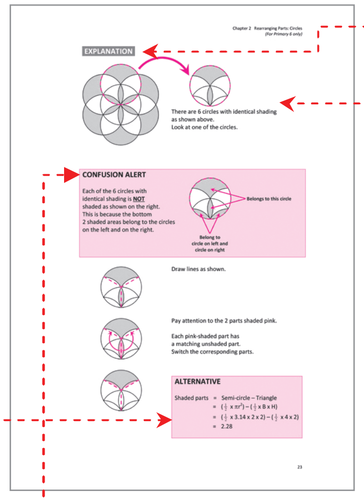
Each example of heuristics application comes with “Working” and “Explanation”, and includes “Confusion Alert” and “Alternative” boxes.

In some chapters, the examples are grouped as follows:

- Non-circles Scenarios (for Primary 5 and Primary 6)
- Circles Scenarios (for Primary 6 only).

WORKING

“Working” shows heuristics application in action (how quick it is to solve a question).



EXPLANATION

"Explanation" shows the thought process (the detailed steps) behind the heuristics application. It takes readers through the solution in the following manner:

- step-by-step method so readers can follow what happens at every stage.
- systematic approach so readers begin to see a pattern in applying the technique.
- easy-to-follow steps so readers can quickly understand the technique minus the frustration.

For Spatial Visualisation, readers will see that its application always involves:

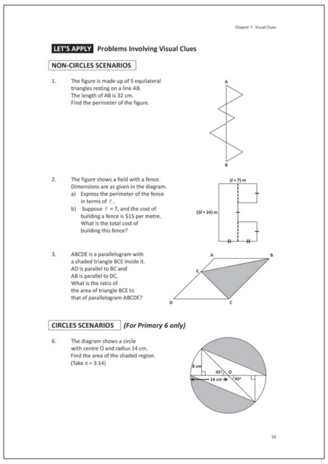
- identifying basic shapes, derivative shapes and parts of shapes,
- analysing how these are inter-related, and how these may be manipulated to arrive at the answer, and
- the application of formulae (as listed in the "Before You Begin" Chapter) to carry out the actual solution.

This quickly helps readers see and understand how to approach each question, including picking out hints often provided in the questions.

"Confusion Alert" boxes highlight areas where students are likely to falter or make mistakes in. It also gives the rationale to help clarify their doubts.

"Alternative" boxes show other approaches to the solution process. This acknowledges the different views that students may have to the problem.

LET'S APPLY



Learning is only effective with practice. Hence, at the end of each chapter/section is a list of questions to hone readers' skills in the heuristics technique.

Where applicable, questions are grouped as follows:

- Non-circles Scenarios (for Primary 5 and Primary 6)
- Circles Scenarios (for Primary 6 only).

ADDITIONAL TIPS

For on-going sharing and discussions on the use of Spatial Visualisation, visit:
www.facebook.com/mathsheuristics

For detailed workings to all the Spatial Visualisation "Let's Apply" sections, visit:
www.mathsheuristics.com/?page_id=472

BEFORE YOU BEGIN

THINGS TO NOTE

1. Where applicable, problems are grouped into Non-circles and Circles Scenarios.
   - **Non-circles Scenarios** are for **Primary 5 and Primary 6**.
   - **Circles Scenarios** are for **Primary 6 only**.

Chapter 2	Completely involves Circles Scenarios, hence entire chapter is for Primary 6 only.
Chapters 3, 5, 6 and 7	Problems in Examples and Let's Apply sections are grouped into Non-Circles and Circles Scenarios.

2. You need to be familiar with the various formulae for different **basic shapes** – rectangle, square, triangle, circle.
   However, the topic on circles is only covered at Primary 6.

3. These formulae will also be needed when working with **derivative shapes** – semi-circle (half circle), quadrant (quarter circle), etc.

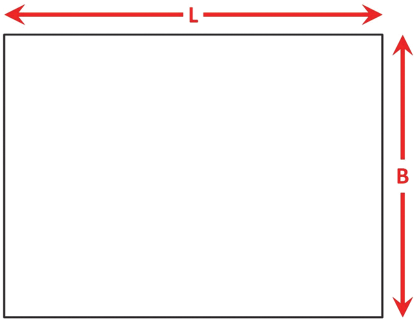
4. Challenging higher-order questions involve a **combination of shapes**.
   The combination sometimes forms a larger **composite shape**.

Examples of **combination of shapes**.
- A semi-circle, a $\frac{1}{4}$ circle and a triangle within a square.
- A square within a circle.

Examples of **composite shape**.
- 2 triangles, forming a square.
- Various squares and rectangles of different sizes, forming a square.

4. **Basic shapes** and their **formulae**.

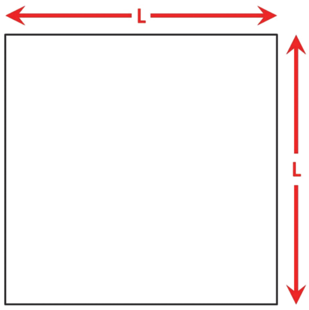
a) **Rectangle**



Area = Length x Breadth
= L x B

Perimeter = 2 Length + 2 Breadth
= 2L + 2B
= 2 (L + B)

b) **Square**

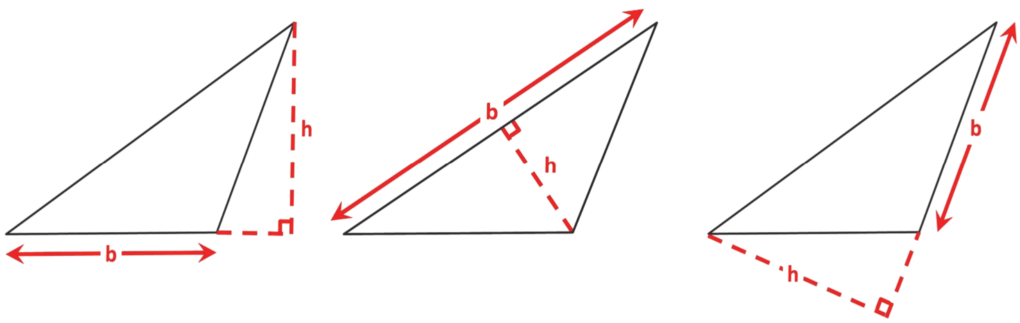


Length = Breadth

Area = Length x Length
= L x L

Perimeter = 4 x Length
= 4 x L

c) **Triangle**

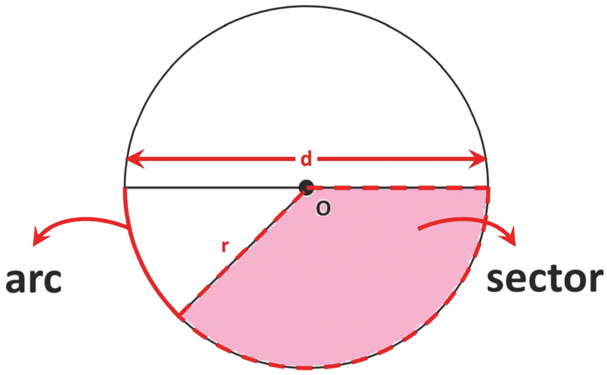


Area = $\frac{1}{2}$ x Base x Height
= $\frac{1}{2}$ x b x h

Perimeter = Sum of 3 sides

($\frac{1}{2}$ bh)

d) **Circle**



Diameter	=	2 x radius	(2r)
Circumference	=	π x diameter	(πd)
	=	2 x π x radius	(2πr)
Area	=	π x radius x radius	(πr^2)

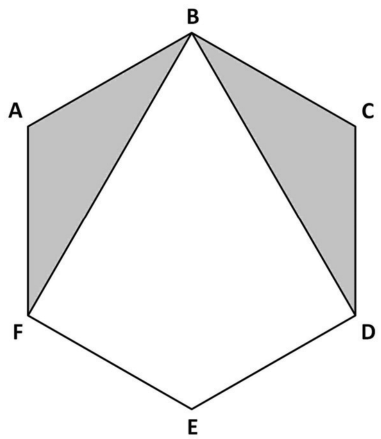
π can assume various forms.
Read the question carefully
to determine which form of π to apply.

- π = 3.14
- π = $\frac{22}{7}$
- Calculator π

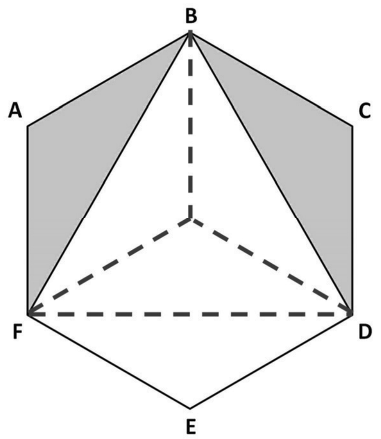
CHAPTER 1 REARRANGING PARTS: NON-CIRCLES

EXAMPLES

1. **In the figure below, AB = BC = CD = DE = EF = FA.**
The area of the whole figure is 120 cm².
Find the area of the shaded part.



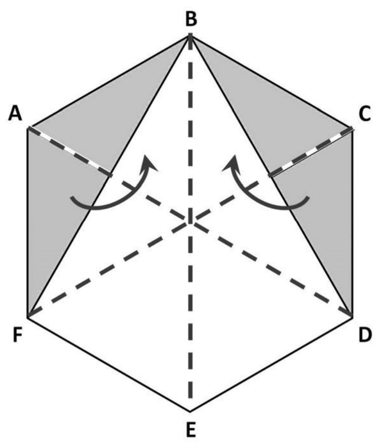
WORKING #1



6 parts = 120
2 parts shaded = $\frac{2}{6}$ x 120
= $\frac{1}{3}$ x 120
= 40

Area of the shaded part is <u>40cm^2</u>.

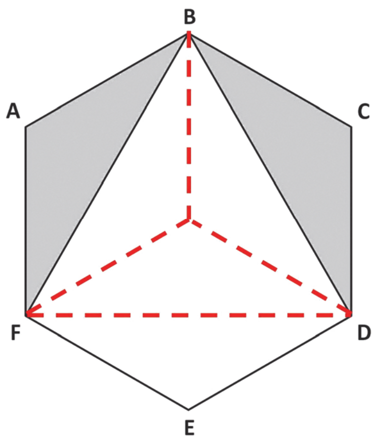
WORKING #2



6 parts = 120
2 parts shaded = $\frac{2}{6}$ x 120
= $\frac{1}{3}$ x 120
= 40

Area of the shaded part is <u>40cm^2</u>.

EXPLANATION #1

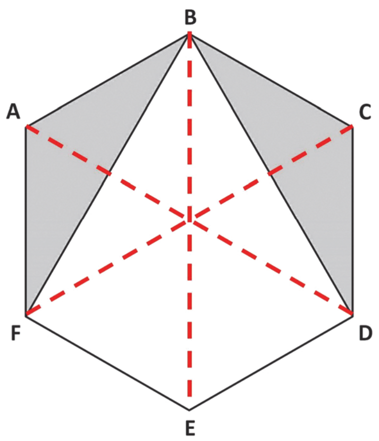


Draw lines as shown.
6 identical triangles are formed.

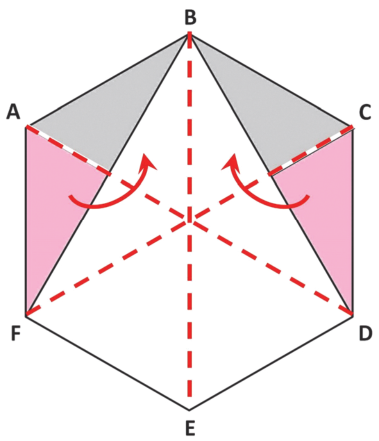
6 identical triangles = 120
2 identical triangles shaded = $\frac{2}{6}$ x 120
= $\frac{1}{3}$ x 120
= 40

Area of the shaded part is 40 cm^2.

EXPLANATION #2

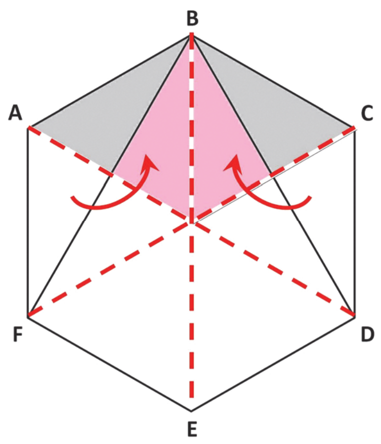


Draw lines as shown.
Looking at the newly-drawn lines,
6 identical segment triangles are formed.



Pay attention to the 2 parts shaded pink.

Each pink-shaded part has
a matching unshaded part.
Switch the corresponding parts.

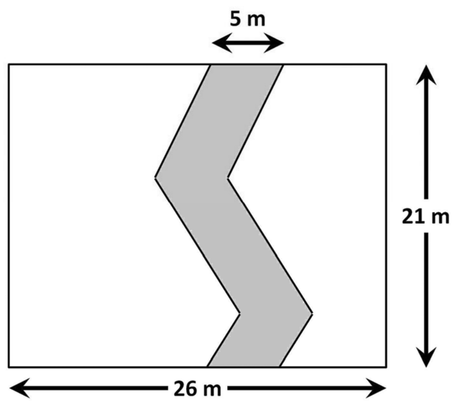


2 of the 6 identical segment triangles are shaded.

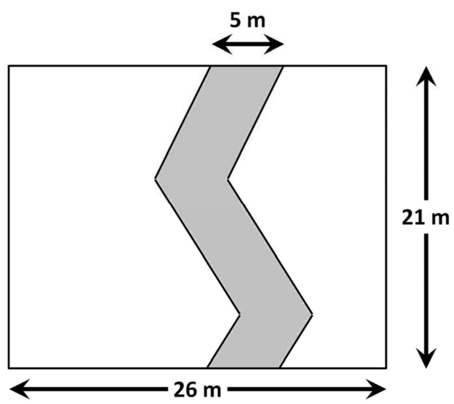
6 identical triangles	=	120
2 identical triangles shaded	=	$\frac{2}{6}$ x 120
	=	$\frac{1}{3}$ x 120
	=	40

Area of the shaded part is <u>40 cm^2</u>.

2. A rectangular garden measuring 26 m by 21 m has a 5-m walking path running through it. Find the area of the walking path.



WORKING



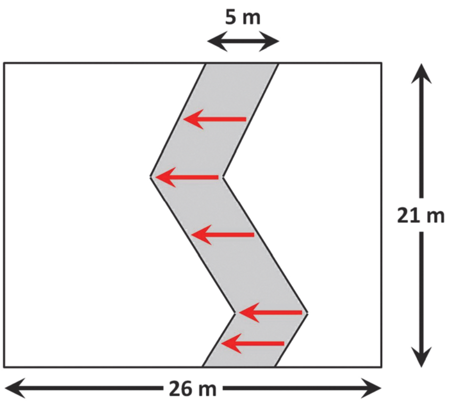
Garden and path = L x B
= 26 x 21
= 546

Garden only = L x B
= (26 – 5) x 21
= 21 x 21
= 441

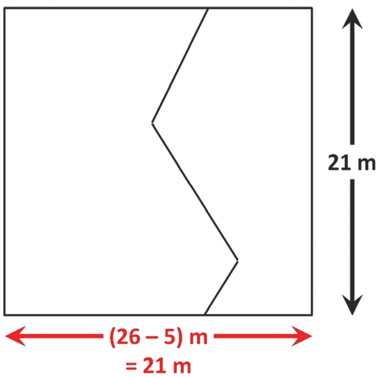
Path = 546 – 441
= 105

Area of the walking path is <u>105 m^2</u>

EXPLANATION



Remove the path and join the 2 garden parts.



This forms a shorter rectangle measuring (26 – 5) m by 21 m, that is 21 m by 21 m.

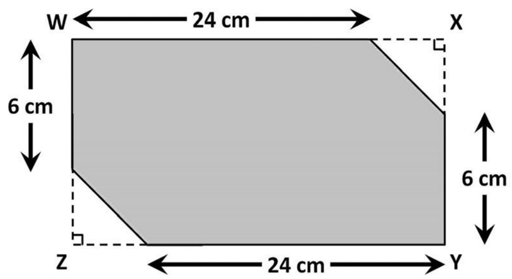
Longer rectangle = L x B
= 26 x 21
= 546

Shorter rectangle = L x B
= 21 x 21
= 441

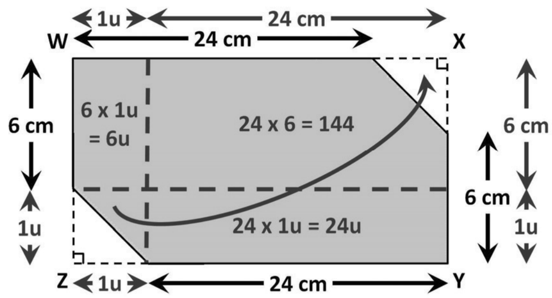
Path = Longer rectangle – Shorter rectangle
= 546 – 441
= 105

Area of the walking path is <u>105 m^2</u>.

3. **The figure below shows a rectangle WXYZ with two identical right-angled isosceles triangles cut out at diagonally-opposite corners. The area of the shaded figure is 264 cm². Find the area of one of the triangles.**



WORKING



Rectangle 1 = 6 x 1u
= 6u

Rectangle 2 = 24 x 1u
= 24u

Rectangle 3 = 24 x 6
= 144

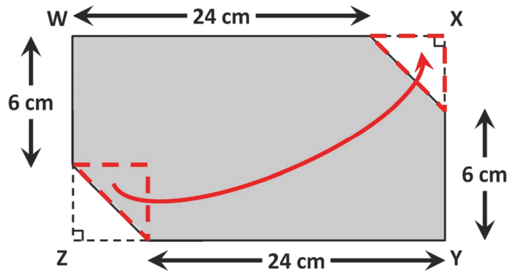
Shaded parts = 144 + 24u + 6u
= 144 + 30u

144 + 30u = 264
30u = 264 – 144
30u = 120
1u = 4

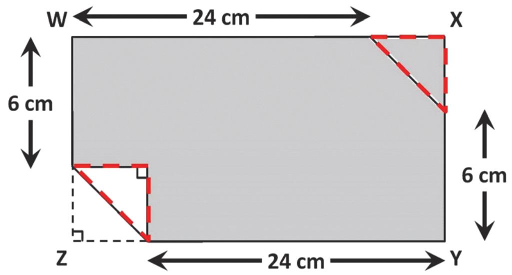
Unshaded triangle = $\frac{1}{2}$ x B x H
= $\frac{1}{2}$ x 1u x 1u
= $\frac{1}{2}$ x 4 x 4
= 8

Area of one of the triangles is 8 cm².

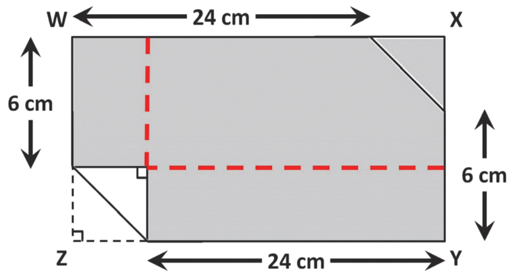
EXPLANATION



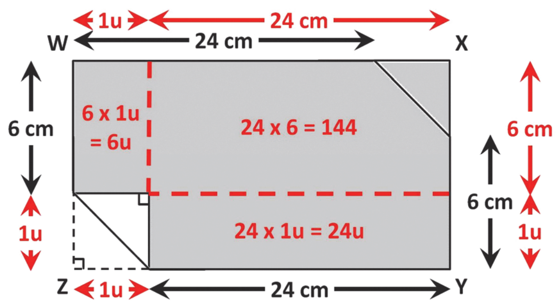
Shift one triangle so that both triangles join to form a square.



This is what you will get.



Draw lines as shown.
There are 3 shaded rectangles.



Rectangle 1	=	6 x 1u
	=	6u
Rectangle 2	=	24 x 1u
	=	24u
Rectangle 3	=	24 x 6
	=	144
Shaded parts	=	144 + 24u + 6u
	=	144 + 30u

We know that the shaded area is 264 cm^2.

Equate both values.

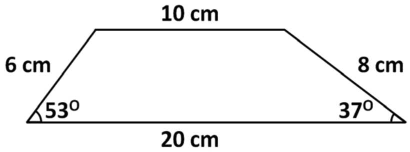
144 + 30u	=	264
30u	=	264 – 144
30u	=	120
1u	=	4

Now, calculate the area of one unshaded triangle.

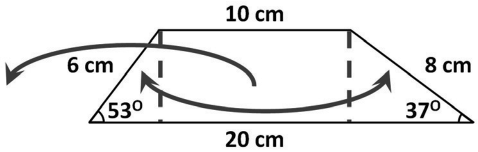
Unshaded triangle	=	$\frac{1}{2}$ x B x H
	=	$\frac{1}{2}$ x 1u x 1u
	=	$\frac{1}{2}$ x 4 x 4
	=	8

Area of one of the triangles is <u>8 cm^2</u>.

4. Find the area of the trapezium shown in the figure below.



WORKING



The 2 triangles
= $\frac{1}{2}$ x B x H
= $\frac{1}{2}$ x 6 x 8
= 24

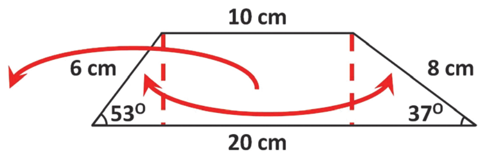
Base of the
2 triangles together
= 20 – 10
= 10
= Base of rectangle

Rectangle
= 2 x 2 triangles
= 2 x 24
= 48

Trapezium
= The 2 triangles
+ Rectangle
= 24 + 48
= 72

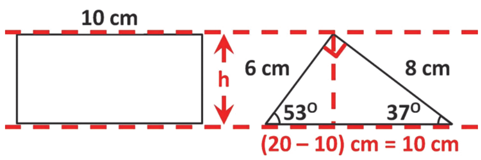
Area of the trapezium is <u>72 cm^2</u>.

EXPLANATION



Draw lines as shown.

Separate the rectangle, and join the 2 triangles.



There are now 2 figures – the rectangle and a larger triangle.

Let's work on the larger triangle.

In the larger triangle, the angle at the top is $180^\circ - 53^\circ - 37^\circ$, that is 90°.

So, B = 6 and H = 8.

Larger triangle
$= \frac{1}{2} \times B \times H$
$= \frac{1}{2} \times 6 \times 8$
$= 24$

Let's work on the rectangle.

Larger triangle $= \frac{1}{2} \times B \times H$
$= \frac{1}{2} \times 10 \times h$
$= 5 \times h$

We already know that the larger triangle is 24 cm^2.

$5 \times h = 24$
$h = 24 \div 5$
$= 4.8$

Rectangle $= B \times H$
$= 10 \times 4.8$
$= 48$

Let's combine.

Trapezium
= Larger triangle + Rectangle
= 24 + 48
= 72

Area of the trapezium is <u>72 cm^2</u>.

ALTERNATIVE

Length of the rectangle
= Base of the larger triangle = 10

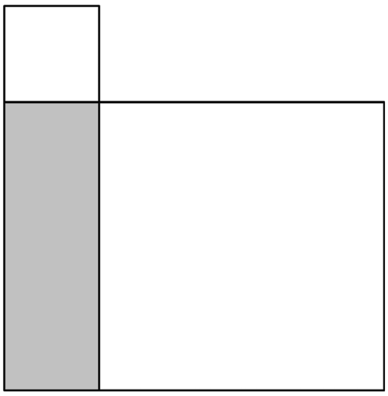
Breadth of the rectangle
= Height of the larger triangle = h

Larger triangle $= \frac{1}{2} \times B \times H$
$= \frac{1}{2} \times 10 \times h$

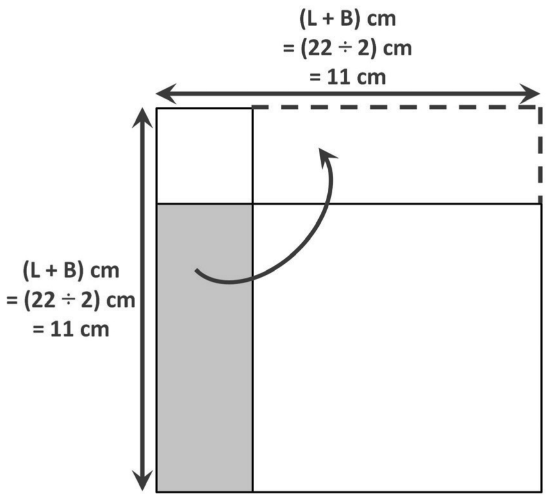
Rectangle $= L \times B$
$= 10 \times h$

So, rectangle = 2 x Larger triangle
= 2 x 24
= 48

5. **The figure below is made up of a small square, big square and a rectangle. The perimeter of the shaded rectangle is 22cm and the sum of the area of the squares is 78.5cm².**
Find the area of the shaded rectangle.



WORKING



Side of composite square
= (22 ÷ 2)
= 11

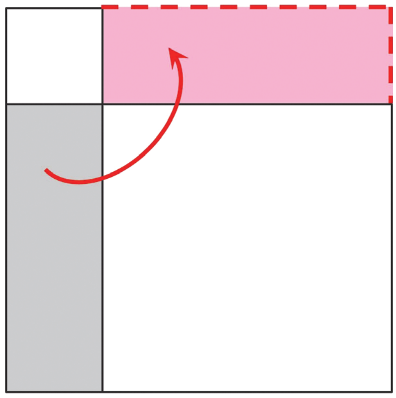
Composite square
= 11 x 11
= 121

2 rectangles
= 121 – 78.5
= 42.5

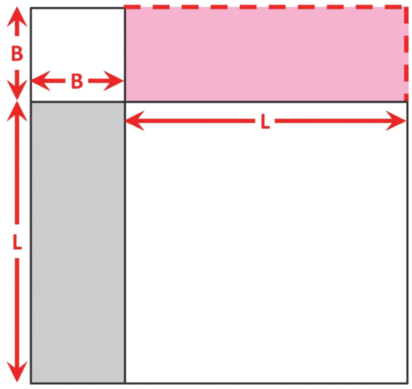
1 rectangle
= 42.5 ÷ 2
= 21.25

Area of the shaded rectangle is 21.25 cm^2.

EXPLANATION



Draw lines as shown.
Note that the 2 rectangles are identical, and that the 4 shapes form a composite square.

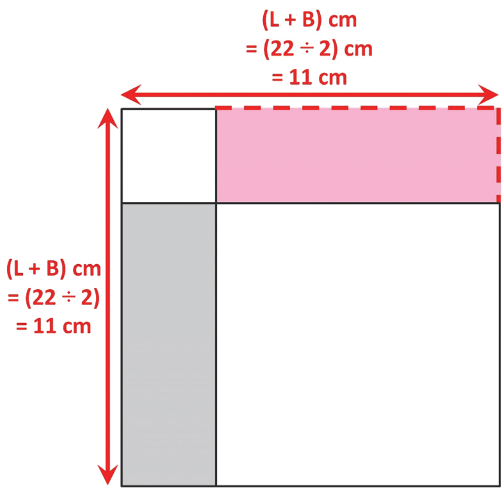


Let's look at the composite square.

Side of composite square
= Length of rectangle
+ Side of small square
= Length of rectangle
+ Breadth of rectangle
= L + B

We know that perimeter of the rectangle is 22 cm or 2 (L + B).

Equate the values of the perimeter.
2 (L + B) = 22
(L + B) = 11



So, side of composite square
= L + B
= 11

Composite square
= 11 x 11
= 121

Now, let's look at the rectangles.

Original 2 squares = 78.5

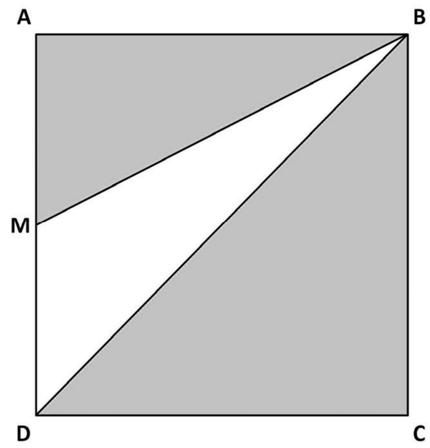
2 rectangles
= Composite square – Original 2 squares
= 121 – 78.5
= 42.5

2 rectangles = 42.5
1 rectangle = 42.5 ÷ 2
= 21.25

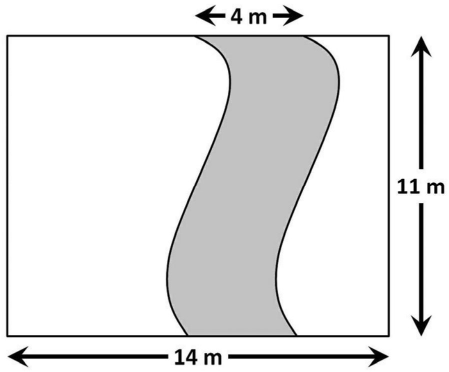
Area of the shaded rectangle is 21.25 cm^2.

LET'S APPLY Problems Involving Rearranging Parts: Non-Circles

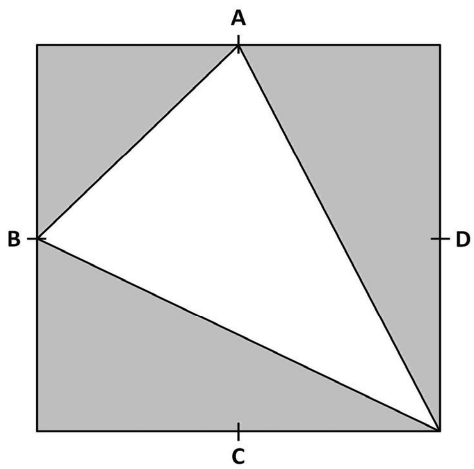
1. A square ABCD is cut into 3 triangles. The area of the square is 80 cm^2. Given that M is the mid-point of AD, find the area of the shaded figure.



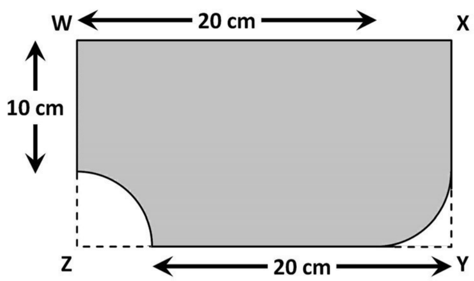
2. A rectangular plot of land measuring 14 m by 11 m has a 4-m walking path running through it. Find the area of the walking path.



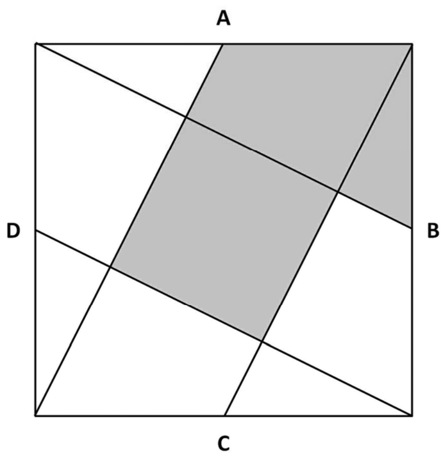
3. A, B, C and D are mid-points of the sides of a square as shown. If the area of the square is 96 cm^2, what is the area of the shaded triangle?



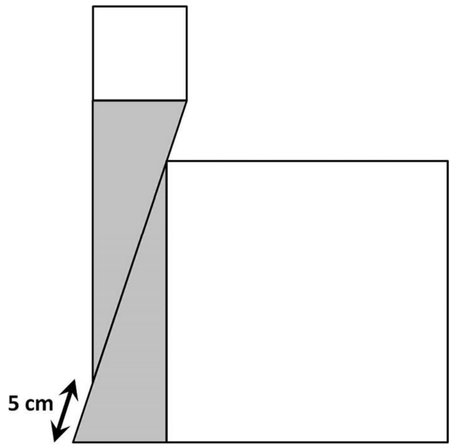
4. The figure shows a rectangle WXYZ. Some parts of it have been cut off, resulting in the shaded shape. The area of the shaded shape is 260 cm^2. Find the area of rectangle WXYZ.



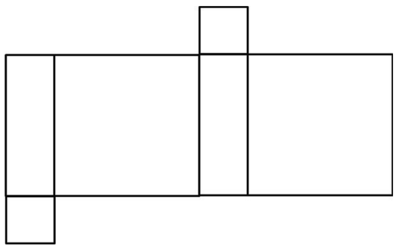
5. The figure shows a big square of 90 cm² area. A, B, C and D are mid-points along the sides of the square.
Find the area of the shaded parts.



6. The figure shows a big square, two identical right-angled triangles and a small square. The perimeter of the shaded region is 44 cm, and the total area of the two squares is 129 cm^2.
Find the area of one triangle.



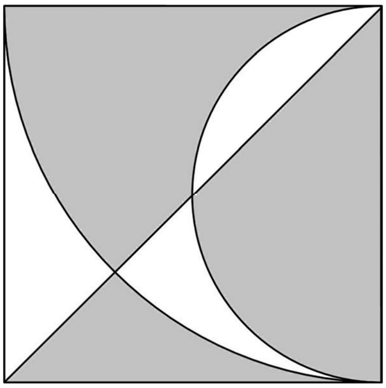
7. The figure consists of two rectangles, two big squares and two small squares. The total area of the four squares is 480 cm². The perimeter of each rectangle is 40 cm.
Find the area of one rectangle.



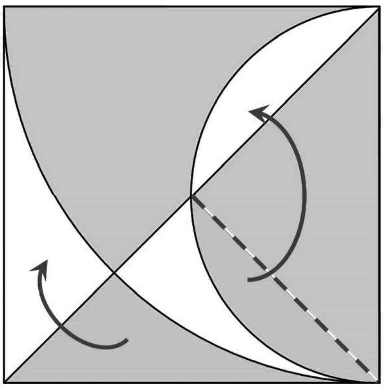
CHAPTER 2 REARRANGING PARTS: CIRCLES *(For Primary 6 only)*

EXAMPLES

1. **The figure below shows a square, a quadrant and a semi-circle.
The area of the square is 240 cm². Find the area of the shaded parts.
(Take π = 3.14)**



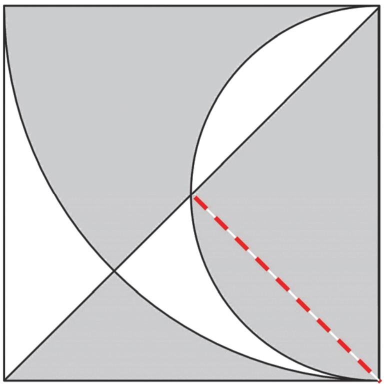
WORKING



$\frac{3}{4}$ square = $\frac{3}{4}$ x 240
= 180

Area of the shaded parts is <u>180 cm²</u>.

EXPLANATION

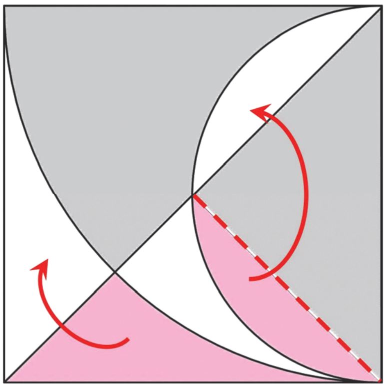


HINT

Clues are always in the question itself. Focus only on shapes mentioned in the question.

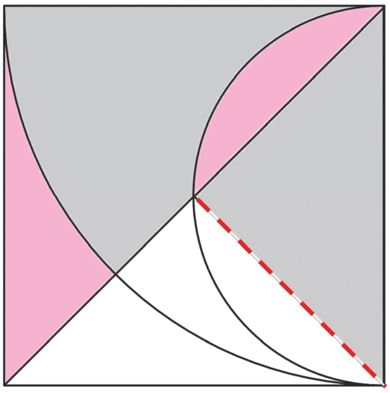
In this example, we can see triangles.
However, since the question does not mention anything about triangles,
we ignore the triangles.
Instead, we focus on the square,
quadrant and semi-circle
which are mentioned in the question.

Draw half a diagonal line as shown.

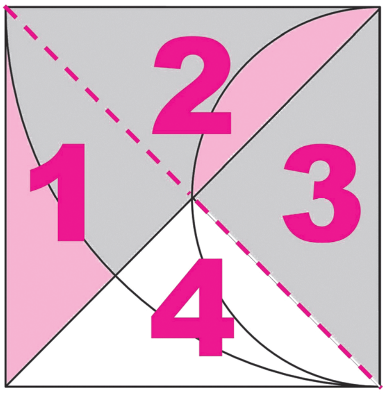


Pay attention to the 2 parts shaded pink.

Each pink-shaded part has
a mirror unshaded part.
Switch the corresponding parts.



This is what you will get.



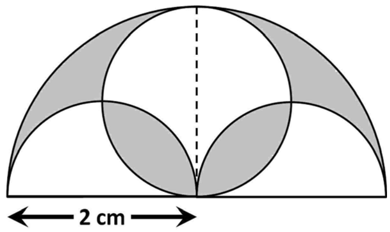
Extend the half diagonal into a full diagonal.
The square is now cut into 4 equal parts.

3 parts are shaded, while 1 part is unshaded.
$\frac{3}{4}$ of the square is shaded.

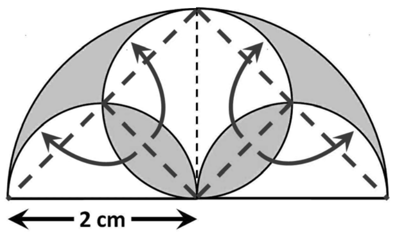
Whole square	=	240
$\frac{3}{4}$ square	=	$\frac{3}{4}$ x 240
	=	180

Area of the shaded parts is <u>180 cm²</u>.

2. The figure below shows a big semi-circle, with a circle and 2 identical smaller semi-circles within it. Find the area of the shaded parts. (Take π=3.14)



WORKING #1

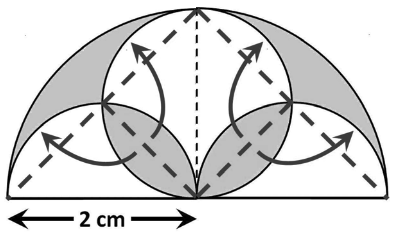


$\frac{1}{2}$ shaded parts = Quadrant – Triangle
= $(\frac{1}{4} \times \pi r^2) - (\frac{1}{2} \times B \times H)$
= $(\frac{1}{4} \times 3.14 \times 2 \times 2) - (\frac{1}{2} \times 2 \times 2)$
= 1.14

Shaded parts = 1.14 x 2
= 2.28

Area of the shaded parts is <u>2.28 cm²</u>.

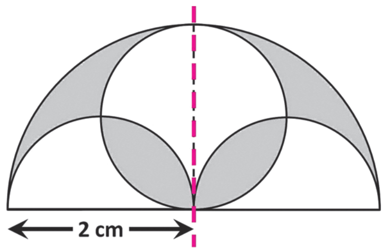
WORKING #2



Shaded parts = Semi-circle – Triangle
= $(\frac{1}{2} \times \pi r^2) - (\frac{1}{2} \times B \times H)$
= $(\frac{1}{2} \times 3.14 \times 2 \times 2) - (\frac{1}{2} \times 4 \times 2)$
= 2.28

Area of the shaded parts is <u>2.28 cm²</u>.

EXPLANATION

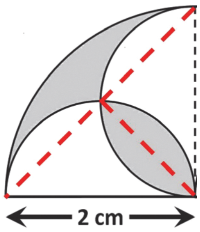


HINT

Clues are always in the question itself. Focus only on shapes mentioned in the question.

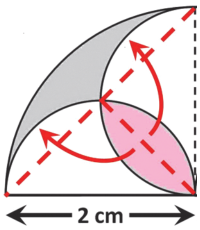
In this example, we focus on the big semi-circle, circle and 2 identical smaller semi-circles which are mentioned in the question.

Note how the semi-circle is made up of 2 quadrants which are mirror image of each other.



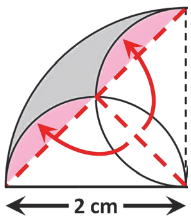
Let’s look at one of the quadrants.

Draw lines as shown.



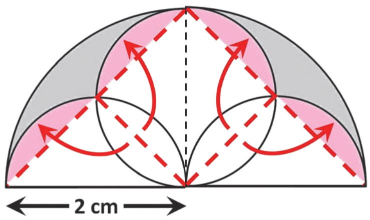
Pay attention to the 2 parts shaded pink.

Each pink-shaded part has
a matching unshaded part.
Switch the corresponding parts.



This is what you will get.

Shaded parts = Quadrant – Triangle
= $(\frac{1}{4} \times \pi r^2) - (\frac{1}{2} \times B \times H)$
= $(\frac{1}{4} \times 3.14 \times 2 \times 2) - (\frac{1}{2} \times 2 \times 2)$
= 1.14



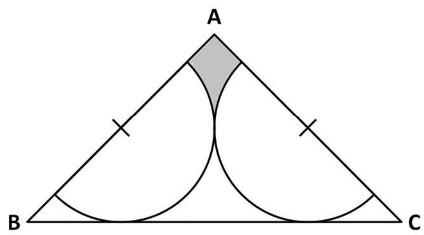
Remember there are 2 identical quadrants.
Total shaded parts = 1.14 x 2
= 2.28

ALTERNATIVE

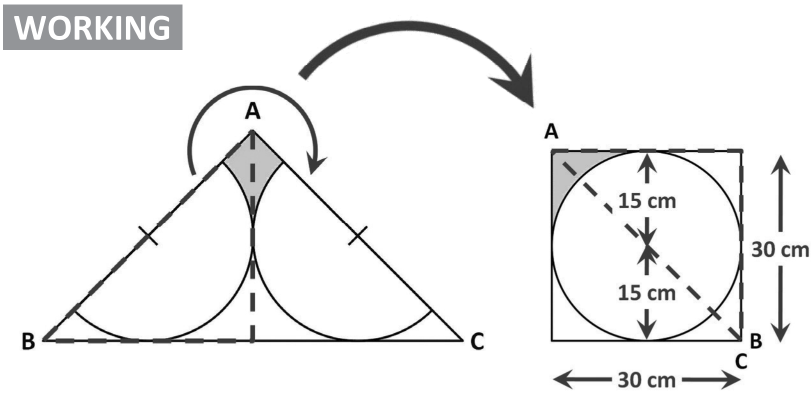
Shaded parts = Semi-circle – Triangle
= $(\frac{1}{2} \times \pi r^2) - (\frac{1}{2} \times B \times H)$
= $(\frac{1}{2} \times 3.14 \times 2 \times 2) - (\frac{1}{2} \times 4 \times 2)$
= 2.28

Area of the shaded parts is <u>2.28 cm²</u>.

3. The diagram below shows an isosceles triangle ABC with 2 semi-circles within it. The semi-circles have a radius of 15 cm. Find the area of the shaded part. (Take π as 3.14)



WORKING



Square = L x B
= 30 x 30
= 900

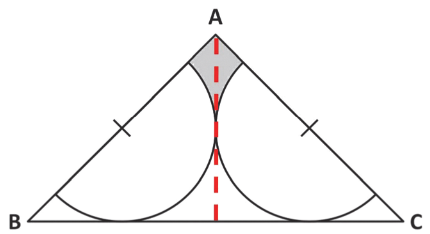
Circle = πr^2
= (3.14 x 15 x 15)
= 706.5

4 corners = Square – Circle
= 900 – 706.50
= 193.50

1 corner = $\frac{1}{4}$ x 193.50
= 48.375

The shaded area is 48.375 cm^2.

EXPLANATION

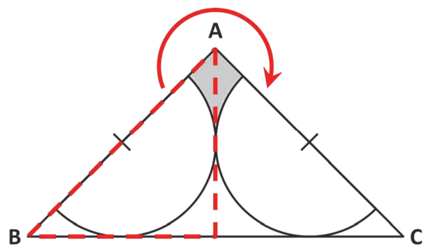


HINT

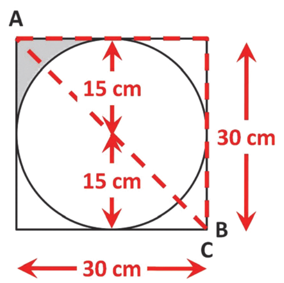
Clues are always in the question itself. Focus only on shapes mentioned in the question.

In this example, we focus on the triangle and 2 semi-circles which are mentioned in the question.

Note how the triangle is made up of 2 smaller triangles which are mirror image of each other.



Rotate the left smaller triangle clockwise on Point A, until Point B touches Point C.

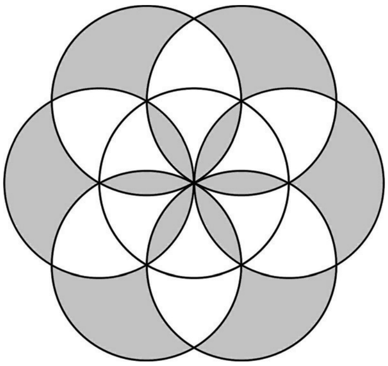


This is what you will get.

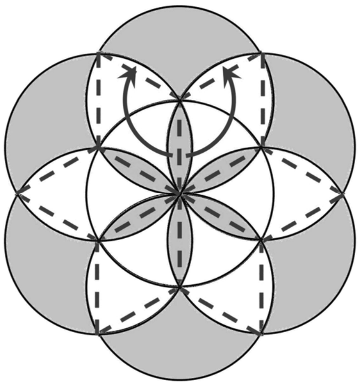
Square	=	L x B
	=	30 x 30
	=	900
Circle	=	πr^2
	=	(3.14 x 15 x 15)
	=	706.5
4 corners	=	Square – Circle
	=	900 – 706.50
	=	193.50
1 corner	=	$\frac{1}{4}$ x 193.50
	=	48.375

The shaded area is <u>48.375 cm^2</u>.

4. **The figure below is formed by overlapping 7 circles of 28 cm diameter. The inner circle passes through the centre of the 6 outer circles. Find the area of the shaded region. (Take $\pi = \frac{22}{7}$)**



WORKING



1 circle → $\frac{1}{3}$ circle shaded

6 circles → $\frac{1}{3}$ x 6 circles shaded

= 2 circles shaded

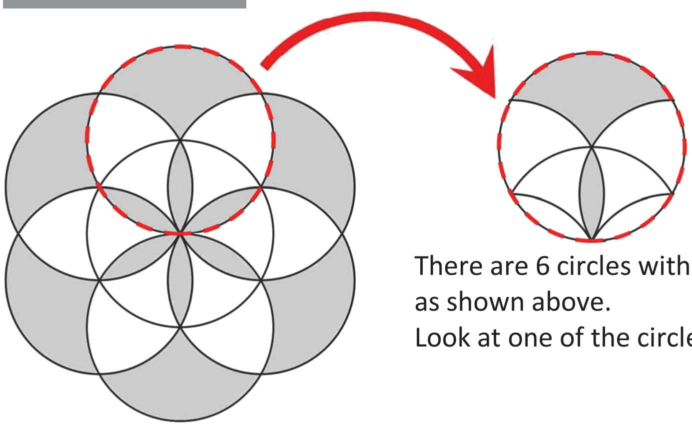
= $2 \times \pi r^2$

= $(2 \times \frac{22}{7} \times 14 \times 14)$

= 1232

Area of the shaded region is <u>1232 cm^2</u>.

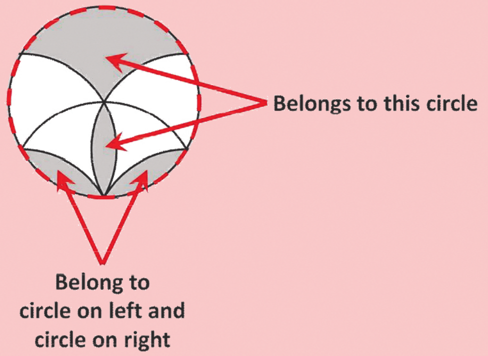
EXPLANATION

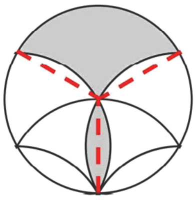


There are 6 circles with identical shading as shown above.
Look at one of the circles.

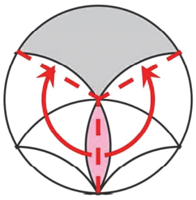
CONFUSION ALERT

Each of the 6 circles with identical shading is **NOT** shaded as shown on the right.
This is because the bottom 2 shaded areas belong to the circles on the left and on the right.



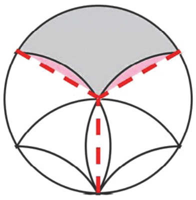


Draw lines as shown.

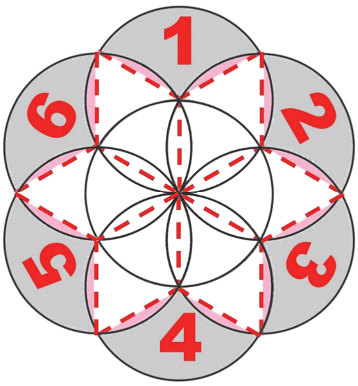


Pay attention to the 2 parts shaded pink.

Each pink-shaded part has a matching unshaded part.
Switch the corresponding parts.



This is what you will get.
$\frac{1}{3}$ of the circle is shaded.

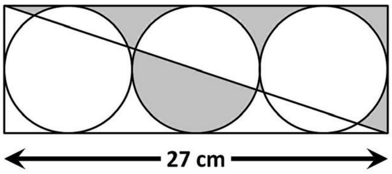


1 circle → $\frac{1}{3}$ circle shaded

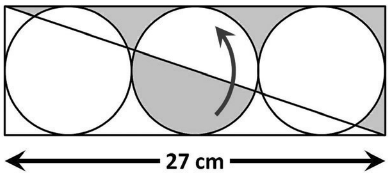
6 circles → $\frac{1}{3}$ x 6 circles shaded
= 2 circles shaded
= 2 x πr^2
= (2 x $\frac{22}{7}$ x 14 x 14)
= 1232

Area of the shaded region is <u>1232 cm^2</u>.

5. The diagram below shows a rectangle and 3 identical circles within it.
Given that the length of the rectangle is 27 cm, find the total area of the shaded parts.
Use a calculator to obtain the value of π. (Leave your answer correct to 2 decimal places)



WORKING

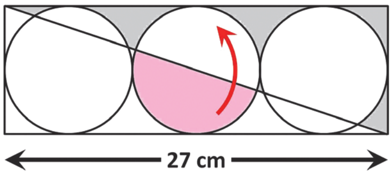


Height of rectangle = Diameter of circle
= 27 ÷ 3
= 9

Shaded area = Triangle – Circle
= $(\frac{1}{2} \times B \times H) - (\pi r^2)$
= $(\frac{1}{2} \times 27 \times 9) - (\pi \times 4.5 \times 4.5)$
= $121.5 - 20.25\pi$
= 57.88

Area of the shaded parts is <u>57.88cm^2</u>.

EXPLANATION

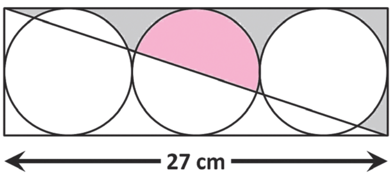


HINT

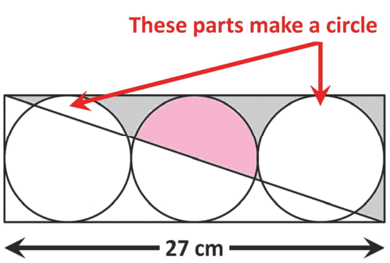
Clues are always in the question itself.
Focus only on shapes mentioned in the question.

In this example, we can see triangles.
However, since the question does not mention anything about triangles,
we ignore the triangles.
Instead, we focus on
the rectangle and 3 circles
which are mentioned in the question.

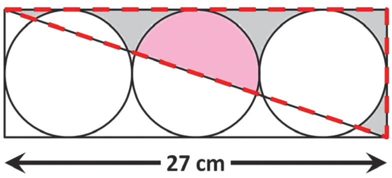
Pay attention to the part shaded pink.
The pink-shaded part has
a matching unshaded part.
Switch the part.



This is what you will get.



The unshaded top segments of
the circles on the left and on the right
form a complete unshaded circle.

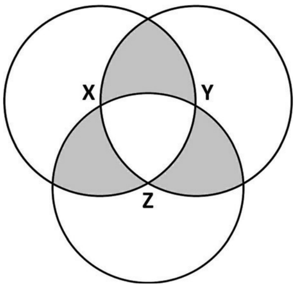


Height of rectangle = Diameter of circle
= $27 \div 3$
= 9

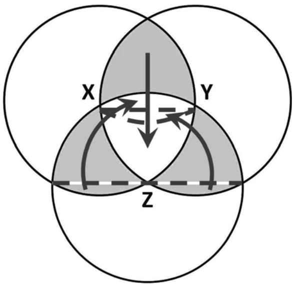
Shaded area = Triangle – Circle
= $(\frac{1}{2} \times B \times H) - (\pi r^2)$
= $(\frac{1}{2} \times 27 \times 9) - (\pi \times 4.5 \times 4.5)$
= $121.5 - 20.25\pi$
= 57.88

Area of the shaded parts is <u>57.88cm^2</u>.

6. The diagram below is made up of 3 circles of 28 cm diameter, with centres, X, Y and Z. Find the area of the shaded regions. (Take $\pi = \frac{22}{7}$)



WORKING



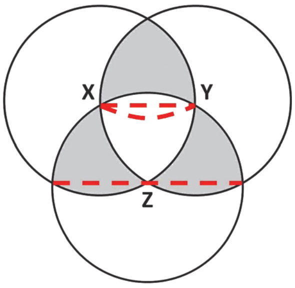
$\frac{1}{2}$ circle $= \frac{1}{2} \times \pi r^2$

$= (\frac{1}{2} \times \frac{22}{7} \times 14 \times 14)$

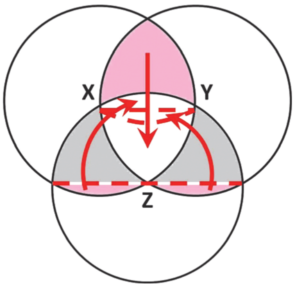
$= 308$

Area of shaded regions is <u>308 cm^2</u>.

EXPLANATION

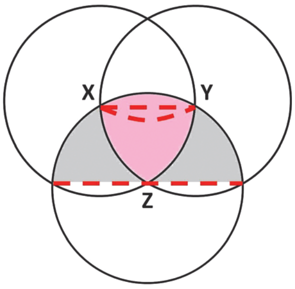


Draw lines shown.



Pay attention to the 3 parts shaded pink.

Each pink-shaded part has
a matching unshaded part.
Switch the corresponding parts.



This is what you will get.
$\frac{1}{2}$ of a circle is shaded.

$\frac{1}{2}$ circle $= \frac{1}{2} \times \pi r^2$

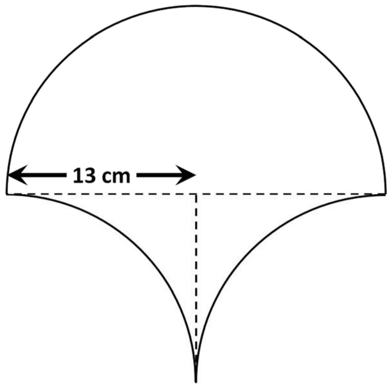
$= (\frac{1}{2} \times \frac{22}{7} \times 14 \times 14)$

$= 308$

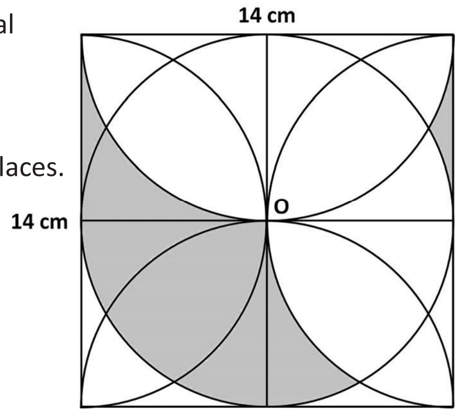
Area of shaded regions is <u>308 cm^2</u>.

LET'S APPLY Problems Involving Rearranging Parts: Circles *(For Primary 6 only)*

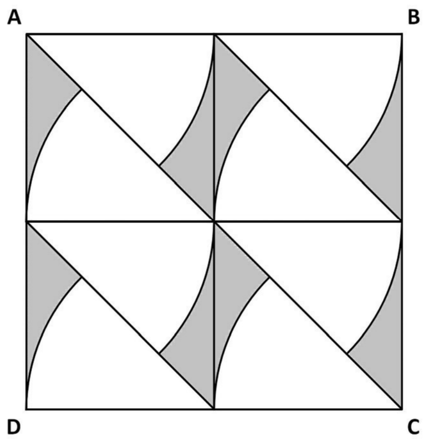
1. The figure shows a window design.
The radius of the semi-circle is 13 cm. Find
a) the perimeter of the figure.
b) the area of the figure.
(Leave your answers correct to 2 decimal places. Take $\pi = 3.14$)



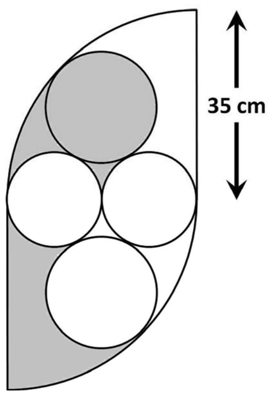
2. The figure is made up of a circle, 4 identical semi-circles and a square of 14 cm sides.
O is the centre of the circle.
Find the area of the shaded regions.
(Leave your answer correct to 2 decimal places.
Take $\pi = 3.14$)



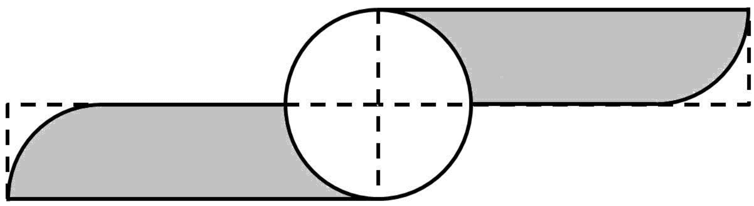
3. The figure shows a big square ABCD with sides measuring 28 cm, and 8 identical sectors.
Find the total area of the shaded parts.
(Take $\pi = \frac{22}{7}$)



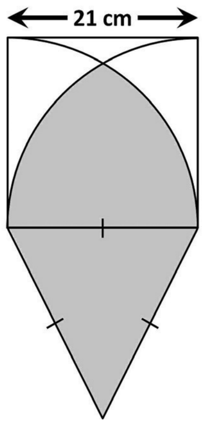
4. The figure is made up of 2 identical small circles, 2 identical medium circles and 2 identical quadrants.
What is the total area of the shaded parts?
(Leave your answer correct to 2 decimal places.
Take $\pi = \frac{22}{7}$)



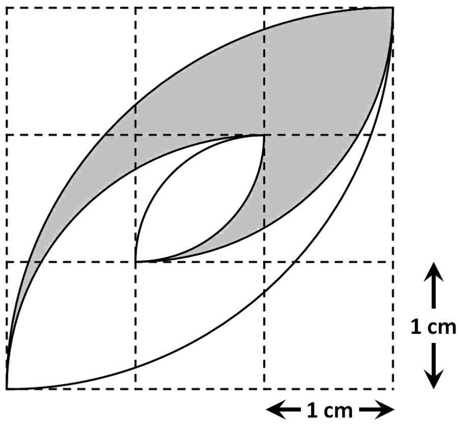
5. The figure below is formed by a circle, and 2 identical rectangles with one end rounded off into a quadrant. Each rectangle has an area of 256 cm^2 before one end is rounded off. The length of the rectangle is 4 times its width. Find the area of the shaded parts. (Take $\pi = \frac{22}{7}$)



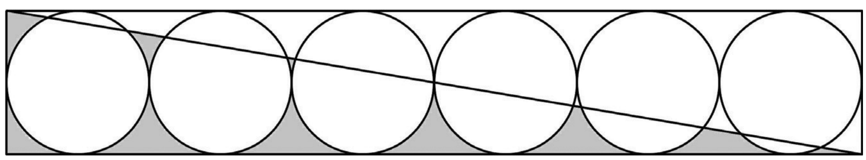
6. The figure shows 2 identical quadrants and an equilateral triangle. Find the area of the shaded parts. (Take $\pi = \frac{22}{7}$)



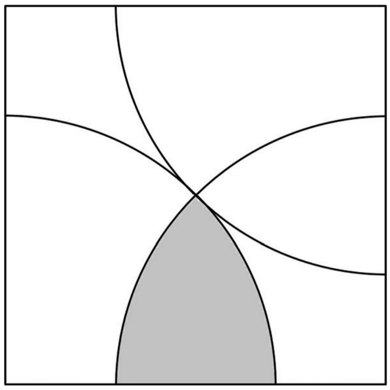
7. The figure shows 2 sets of 3 quadrants with radii of 1 cm, 2 cm and 3 cm. Find the area of the shaded parts. (Take π = 3.14)



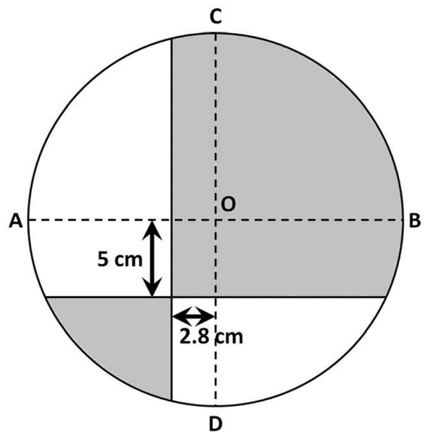
8. The figure below consists of a rectangle and six similar circles of 10 cm radius. Find the area of the shaded regions. (Take π = 3.14)



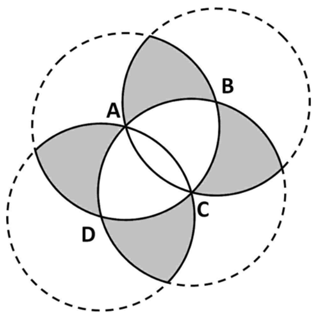
9. The figure is made up of a square and 3 identical quadrant of 28 cm radius. Find the area of the shaded part. (Take $\pi = \frac{22}{7}$)



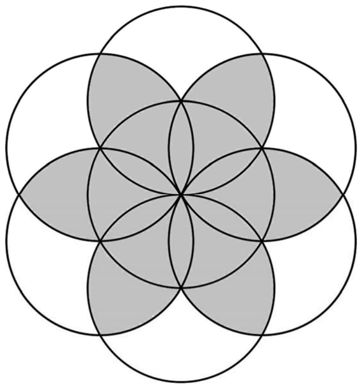
10. The diagram shows a circle with 12 cm radius and centre O. AB and CD are diameters of the circle. Find the area of the shaded parts. (Take π = 3.14)



11. The diagram shows 4 identical circles of 42 cm radius, with centres A, B, C and D. Find the area of the shaded parts. (Take $\pi = \frac{22}{7}$)



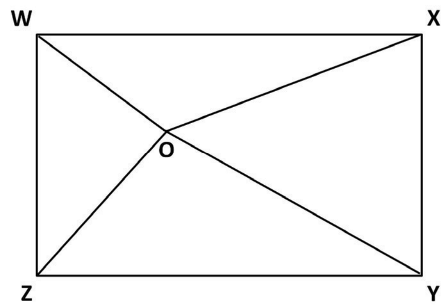
12. The figure is formed by overlapping 7 circles of 14 cm diameter. The inner circle passes through the centres of the 6 outer circles. Find the area of the shaded regions. (Take $\pi = \frac{22}{7}$)



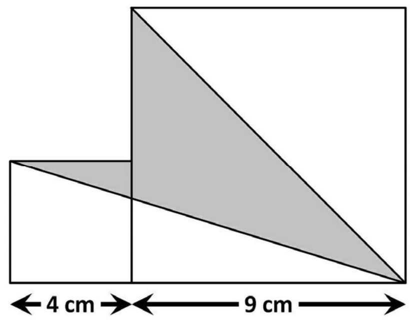
LET'S APPLY Problems Involving Drawing Lines

NON-CIRCLES SCENARIOS

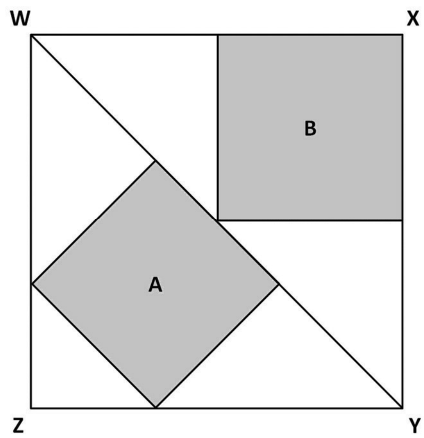
1. A rectangle WXYZ is divided into four triangles. The areas of triangle WZO and triangle YZO are 16 cm² and 27 cm² respectively. The area of triangle XYO to the area of rectangle WXYZ is 3:10. Find the area of the triangle WXO.



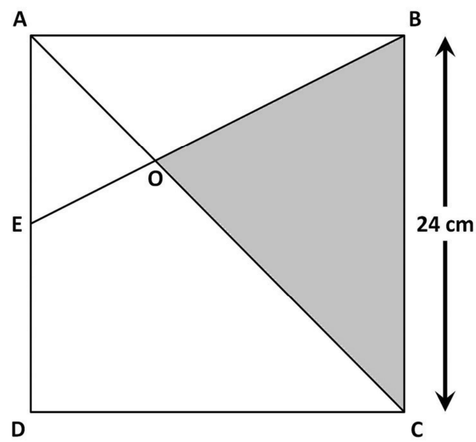
2. The figure is made up of 2 squares of sides 4 cm and 9 cm. Find the area of the shaded figure.



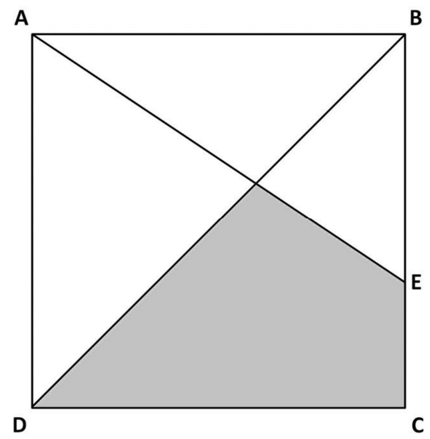
3. WXYZ is a square with an area of 90 cm². The shaded parts A and B are 2 squares with different areas. Find the area of the shaded parts.



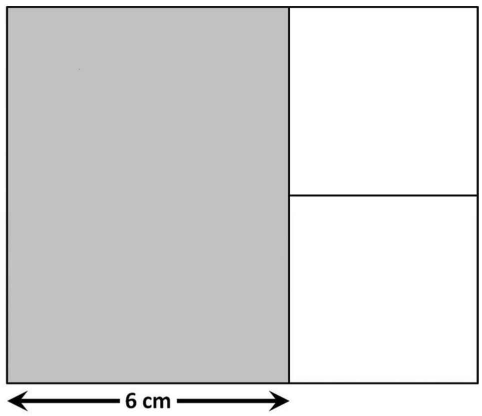
4. ABCD is a square. AOC and BOE are straight lines. Given that E is the mid-point of AD, find the shaded area of the figure.



5. The figure below shows a square ABCD with 30 cm sides. The ratio of lengths BE to EC is 2: 1. Find the area of the shaded region.

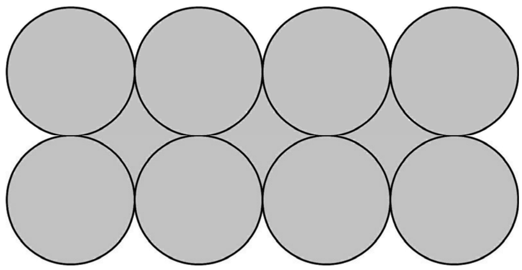


6. The figure is made up of a rectangle and 2 identical squares. The area of the shaded rectangle is thrice the area of a square. Given that the breadth of the rectangle is 6 cm, find the perimeter of the whole figure?

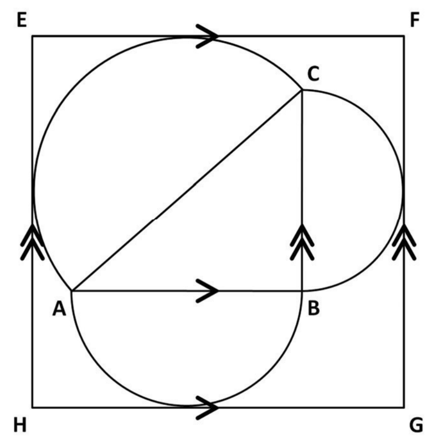


CIRCLES SCENARIOS *(For Primary 6 only)*

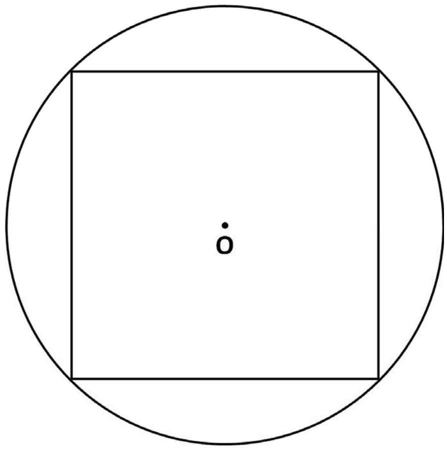
7. The figure shows 8 identical circles of 1 cm radius. Their centres are placed along the lengths of a rectangle. The centres of the 4 end-circles are places at the 4 corners of the rectangle. Find the area of the shaded figure. (Take π = 3.14)



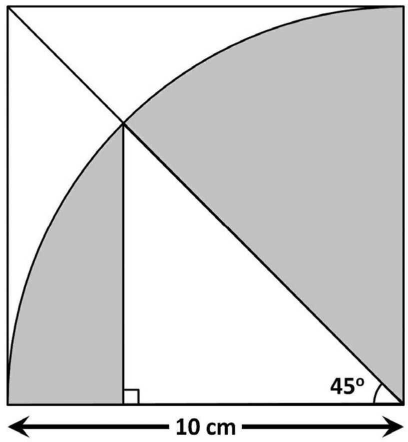
8. The diagram, not drawn to scale, shows a figure EFGH, 3 semi-circles and a right-angled triangle. If AC = 26 cm, BC = 24 cm and AB = 10 cm, find the area of EFGH.



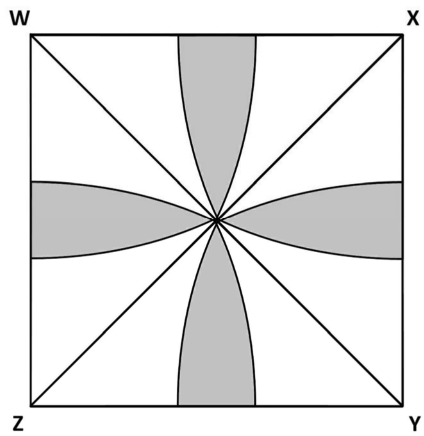
9. A square is drawn in a circle with centre O as shown. The area of the square is 46 cm². Find the area of the circle. (Take π = 3.14)



10. The diagram, not drawn to scale, shows a right-angled triangle and a quadrant. Find the area of the shaded parts. (Take π = 3.14)



11. The figure shows square WXYZ with diagonals measuring 140 cm. Given that the shaded parts are identical, find the total area of the shaded parts. (Take $\pi = \frac{22}{7}$)



Answers to questions in the prior chapters' Let's Apply sections
are listed in this chapter. Detailed workings may be downloaded at:
www.mathsheuristics.com/solutions

CHAPTER 1
REARRANGING PARTS: NON-CIRCLE

LET'S APPLY

Problems Involving Rearranging Parts: Non-circle

1. 60 cm^2
2. 44 m^2
3. 60 cm^2
4. 264 cm^2
5. 36 cm^2
6. 40 cm^2
7. 80 cm^2

ISBN: 978-981-07-8948-0

Answers to questions in the prior chapters' Let's Apply sections
are listed in this chapter. Detailed workings may be downloaded at:
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CHAPTER 2
REARRANGING PARTS: CIRCLES

LET'S APPLY

Problems Involving Rearranging Parts: Circles

1. a) 81.64 cm
   b) 338 cm^2
2. 59.54 cm^2
3. 168 cm^2
4. 735.43 cm^2
5. 384 cm^2
6. 462 cm^2
7. 21.12 cm^2
8. 258 cm^2
9. 224 cm^2
10. 254.08 cm^2
11. 3693 cm^2
12. 308 cm^2

ISBN: 978-981-07-8948-0

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The Mastering Heuristics Series was conceptualised by Sunny Tan, Principal Trainer of mathsHeuristics™, to give parents and students a comprehensive guide to Heuristics. The Series neatly packages Heuristics techniques into a series of guidebooks with well-defined application scenarios. It offers many examples, showing the efficiency and step-by-step application of Heuristics techniques, plus opportunities to get in some practice.

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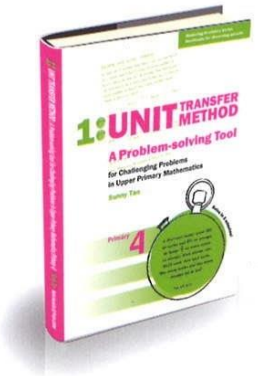
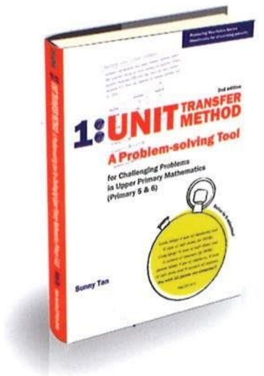
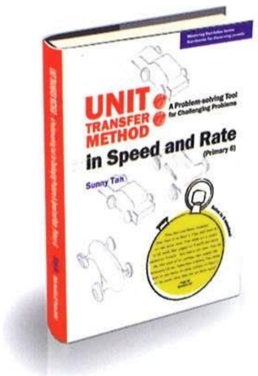


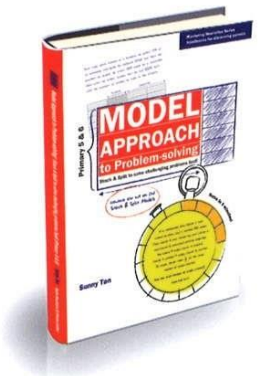
Unit Transfer Method @P4



Unit Transfer Method @P5 & P6



Unit Transfer Method (Speed & Rate) @P6



Model Approach to Problem-solving

Author

Sunny Tan trains students, especially those in the PSLE year, in the use of various Heuristics techniques.
He also conducts Heuristics workshops for parents and educators.

In the 1990s, NIE-trained Sunny taught primary and secondary Maths in various streams. He observed how the transformed primary Maths syllabus stumped children, parents and, sometimes, even teachers. How do you teach young children to accurately choose and sequentially apply different situational logic in solving non-routine problems? Sunny resolved to simplify the learning and application of such skills. After a few years of research and development, Sunny eventually established the mathsHeuristics™ programme – and now the Mastering Heuristics Series – which has achieved consistent success and effectiveness.

Sunny's ingenious methodology has attached much media interest - The Straits Times, The Business Times, The New Paper, TODAY, Shin Min Daily News, FM938 LIVE and major parenting magazines - as well as raving reviews by academia and parents.

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