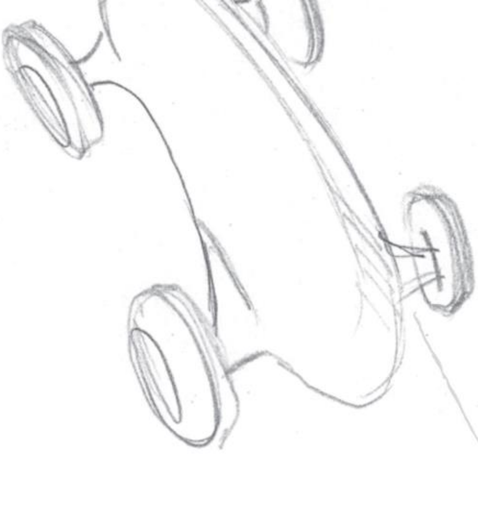
Mastering Heuristics Series
Handbooks for discerning parents

UNIT TRANSFER METHOD in Speed and Rate

A Problem-solving Tool for Challenging Problems

(Primary 6)

Sunny Tan



Solve in 5 minutes!

Tom, Ben and Harry travelled from Point X to Point Y. They left Point X at the same time. Tom drove at a speed of 63 km/h, Ben jogged at 9 km/h and Harry walked at 7 km/h. Ben took a lift from Tom for the first part of his journey, then jogged the remaining 2.8 km. When Ben alighted, Tom drove back to pick Harry. All three arrived at Point Y at the same time. How far did Harry walk?

Page 45
Question 10

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Unit Transfer Method

A Problem-solving Tool
for Challenging Problems
in Speed and Rate
(Primary 6)

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CONTENTS

PREFACE

Heuristics in Primary Maths Syllabus

Heuristics is a specialised mathematical problem-solving concept. Mastering it facilitates efficiency in solving regular as well as challenging mathematical problems. The Ministry of Education in Singapore has incorporated 11 Problem-solving Heuristics into all primary-level mathematics syllabuses.

Learning Heuristics Effectively

However, the 11 Problem-solving Heuristics are not taught systematically; they have been dispersed into the regular curriculum. This not only makes it difficult for students to pick up Heuristics skills, but can also make mathematics confusing for them. For us parents, it is difficult to put aside the regular-syllabus mathematical concepts we were brought up on to re-learn Heuristics, much less teach our own children this new concept.

Take the Heuristic technique of Algebraic Equations, for instance. Although this technique has never been, and is still not, taught at primary level, parents may attempt to teach their children to solve mathematical questions using Algebraic Equations. This will only confuse their children. According to current primary-level mathematics syllabus, other Heuristics techniques should be used instead.

These and other challenges were what I observed firsthand during my years as a mathematics teacher, and provided me the impetus for my post-graduate studies, mathsHeuristics™ programmes and the Mastering Heuristics Series of guidebooks.

About Mastering Heuristics Series

This series of books is a culmination of my systematic thinking and experience, supported by professional instructional writing and editing, to facilitate understanding and mastery of Heuristics. I have neatly packaged Heuristics into main techniques (series of guidebooks) and mathematical scenarios (chapters within each guidebook). For each mathematical scenario, I offer several examples, showing how a particular heuristics technique may be applied, and then explaining the application in easy-to-follow steps and illustrations – without skipping a beat.

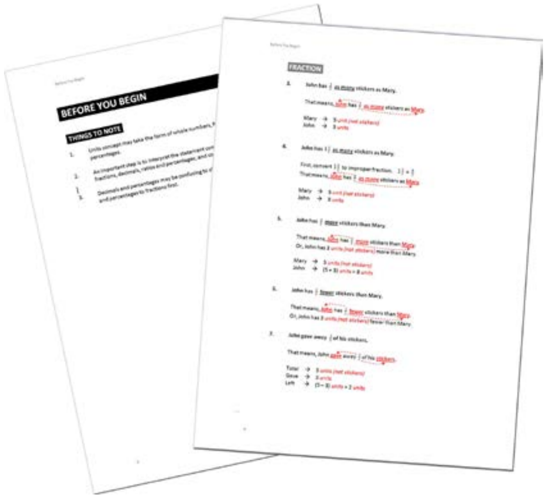
This particular guidebook in the series deals specifically with *Unit Transfer Method in Speed and Rate* – the use of ratio to effectively analyse and solve challenging Speed and Rate problems. This simple, logical yet powerful problem-solving technique complements the model approach.

The Mastering Heuristics Series provides a comprehensive guide to Heuristics. While each guidebook introduces parents to how Heuristics works, students have the opportunity to see the technique applied in different scenarios and to get in some practice. For students enrolled in mathsHeuristics™ programmes, each guidebook serves as a great companion, while keeping parents well-informed of what their children are learning.

Sunny Tan
October 2012

HOW TO USE THIS BOOK

BEFORE YOU BEGIN

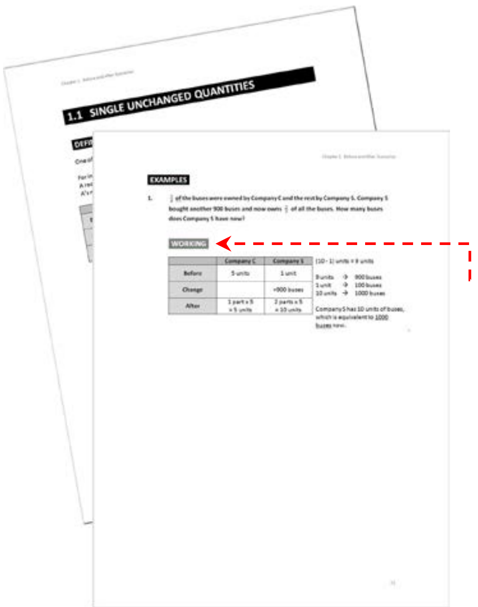


This chapter instills basic but important steps and truths in the heuristics technique that must be applied across every question in the guidebook. This helps to standardise the given information for easy application of the technique being taught.

In this guidebook on Unit Transfer Method (UTM) in Speed and Rate, the steps include the speed formula and UTM rules unique to Speed and Rate scenarios. These build on the basic UTM rules covered in the Mastering Heuristics Series – to convert whole numbers, fractions, decimals, percentages and ratios into units.

CHAPTERS AND SECTIONS

Various scenarios are neatly separated into different chapters and sections. This allows the heuristics technique to be learnt and applied in a focused manner.

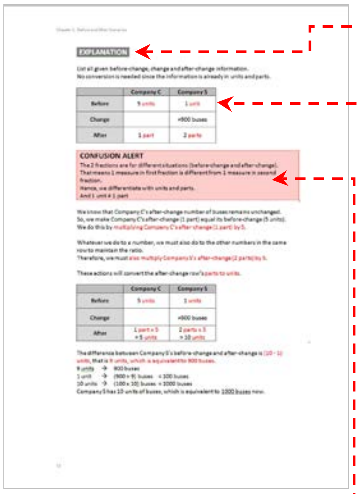


EXAMPLES

Each example of heuristics application comes with “Working” and “Explanation”, and includes “Confusion Alert” and “Alternative” boxes.

WORKING

“Working” shows heuristics application in action (how quick it is to solve a question).



EXPLANATION

"Explanation" shows the thought process (the detailed steps) behind the heuristics application. It takes readers through the solution in the following manner:

- step-by-step method so readers can follow what happens at every stage.
- systematic approach so readers begin to see a pattern in applying the technique.
- easy-to-follow steps so readers can quickly understand the technique minus the frustration.

For UTM in Speed and Rate, readers will see that its application always:

- begins with the UTM basics explained in the "Before You Begin" chapters found in the other guidebooks in the Mastering Heuristics Series, and
- involves the steps and truths explained in the "Before You Begin" chapter of this particular guidebook.

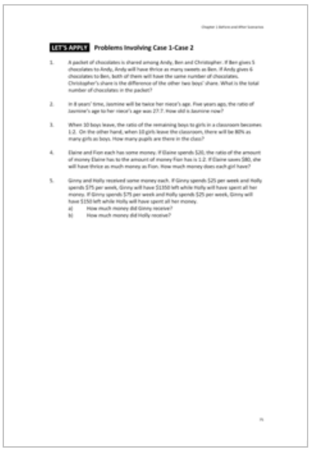
This quickly helps readers see and understand the relationships among all the information given in the question.

"Confusion Alert" boxes highlight areas where students are likely to falter or make mistakes in. It also gives the rationale to help clarify their doubts.

"Alternative" boxes show other approaches to the solution process. This acknowledges the different views that students may have to the problem.

LET'S APPLY

Learning is only effective with practice. Hence, at the end of each chapter/section is a list of questions to hone readers' skills in the heuristics technique.



ADDITIONAL TIPS

For on-going sharing and discussions on the use of UTM in Speed and Rate, visit:
www.facebook.com/mathsheuristics

For detailed workings to all the UTM in Speed and Rate "Let's Apply" sections, visit:
www.mathsheuristics.com/?page_id=472

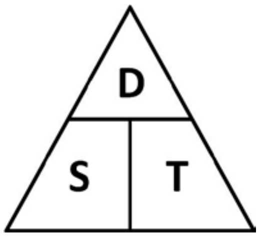
BEFORE YOU BEGIN

THINGS TO NOTE

1. Speed = Distance ÷ Time or $\frac{D}{T}$

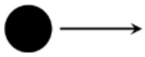
2. Distance = Speed x Time or S x T

3. Time = Distance ÷ Speed or $\frac{D}{S}$



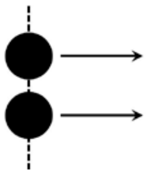
4. Problems involving speed can be generally categorised into four scenarios.

Scenario 1: One object moving

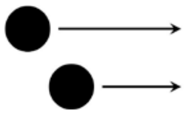


Scenario 2: Two objects (or one object in two different situations) moving in the same direction

From the same starting point.

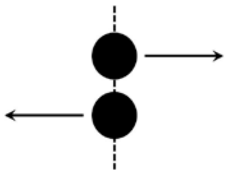


From different starting points.



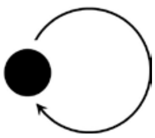
Scenario 3: Two objects moving in opposite directions

From the same starting point.



From different starting points, towards each other.

Scenario 4: Objects (or objects in different situations) moving in circular direction(s)

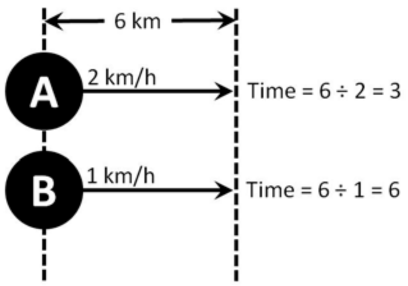


5. Some rules to remember when using Unit Transfer Method (UTM) to solve problems involving Speed.

a) When two objects (or one object in two different situations) travel the **<u>same distance</u>** (with known speeds), the **<u>ratio of speed INVERTED</u>** is the **<u>ratio of time taken</u>**.

ILLUSTRATION

- **<u>Distance same</u>** for both objects / situations.
- Speed known.
- Time unknown.



Speed A	:	Speed B	
2	:	1	
Time A	:	Time B	
3	:	6	← Simplify actual time
1	:	2	← Inversion of speed ratio

Hence, when using UTM
to solve problems involving Speed,
where **<u>distance is the same</u>**,
just tabulate speeds,
simplify speed ratio (if necessary),
then inverse the figures for time ratio.

	A	B
Speed	2	1
Time	1	2

b) Similarly, when two objects (or one object in two different situations) travel the **<u>same distance</u>** (with known time taken), the **<u>ratio of time taken INVERTED</u>** is the **<u>ratio of speed</u>**.

EXAMPLE

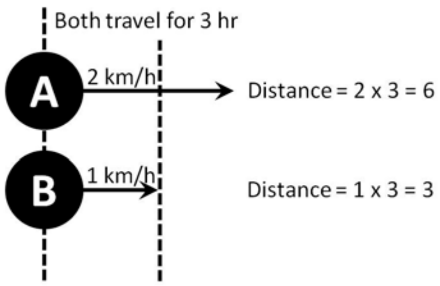
- **<u>Distance same</u>** for both objects / situations.
- Time known.
- Speed unknown.

	A	B
Time	2	1
Speed	1	2

c) When two objects (or one object in two different situations) travel over the **same time** (with known speeds), the **ratio of speed is SAME** as the **ratio of distance travelled**.

ILLUSTRATION

- **Time same** for both objects / situations.
- Speed known.
- Distance unknown.



Speed A	:	Speed B	
2	:	1	
Distance A	:	Distance B	
6	:	3	← Simplify actual distance
2	:	1	← Repeat of speed ratio

Hence, when using UTM to solve problems involving Speed, where **time travelled is the same**, just tabulate speeds, simplify speed ratio (if necessary), then repeat the figures for distance ratio.

	A	B
Speed	2	1
Distance	2	1

d) Similarly, when two objects (or one object in two different situations) travel over the **same time** (with known distance), the **ratio of distance travelled is SAME** as the **ratio of speed taken**.

EXAMPLE

- **Time same** for both objects / situations.
- Distance known.
- Speed unknown.

	A	B
Distance	2	1
Speed	2	1

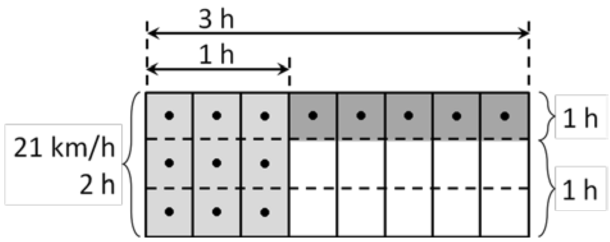
6. Occasionally, there will be Speed and Rate word problems where logical reasoning and/or the Model Approach is required, without the need for Unit Transfer Method in Speed and Rate. For completeness, this guidebook also covers such word problems.

CHAPTER 1 SPEED: ONE OBJECT MOVING

EXAMPLES

1. Richard took 3 hours to cycle from Point A to Point B. He covered $\frac{3}{8}$ of the journey in the first hour, $\frac{1}{3}$ of the remaining journey in next hour, and the rest in the third hour. Given that his average speed for the first two hours was 21 km/h, find his average speed for the entire journey.

WORKING



14u → 42 km
1u → (42 km ÷ 14) = 3 km
24u → (24 x 3 km) = 72 km

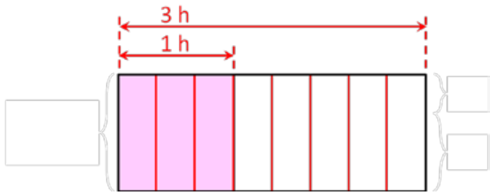
Average speed = (72 km ÷ 3 h)
= 24 km/h

Richard's average speed for the entire journey was 24 km/h.

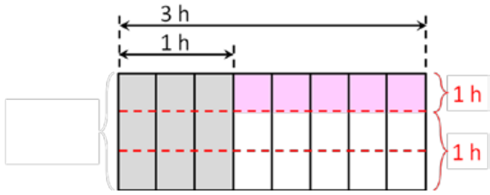
EXPLANATION

This example uses the **<u>Model method</u>**
instead of Unit Transfer Method in Speed and Rate.

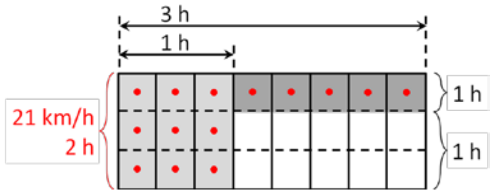
1st hour, covered $\frac{3}{8}$ of the journey.
Draw vertical division lines to make 8 units,
and shade 3 units out of the 8 units.



2nd hour, covered $\frac{1}{3}$ of the remaining journey.
Ignore the vertical division lines,
draw horizontal division lines to make 3 parts,
shade 1 part out of the 3 unshaded parts.



Considering all the division lines,
there are 24 sub-parts.
In the 1st hour and 2nd hour,
14 sub-parts are covered.
We also know that
in the 1st hour and 2nd hour,
the average speed was 21 km/h.
This means distance covered
= Speed x Time = (21 km/h x 2 h) = 42 km



14u	→	42 km		
1u	→	(42 km ÷ 14)	=	3 km
24u	→	(24 x 3 km)	=	72 km

Average speed = (72 km ÷ 3 h)
= 24 km/h

Richard's average speed for the entire journey was <u>24 km/h</u>.

2. **Samuel drove at a certain speed for the first 40 minutes of his journey. Then he increased his speed by 30 km/h for the next 10 minutes. Given that he drove a total of 85 km, find**

a) his driving speed for the first 40 minutes;

b) his driving speed for the last 10 minutes.

WORKING

Speed	Time	Distance
1u	$\frac{2}{3}$	$\frac{2}{3}$ u
1u + 30	$\frac{1}{6}$	$\frac{1}{6}$ u + 5

($\frac{2}{3}$ u + $\frac{1}{6}$ u + 5) → 85

$\frac{5}{6}$ u → 80

a) 1u → (80 km/h x $\frac{6}{5}$) = 96 km/h

b) (96 + 30) km/h = 126 km/h

a) Samuel's driving speed for the first 40 minutes was <u>96 km/h</u>.

b) Samuel's driving speed for the last 10 minutes was <u>126 km/h</u>.

EXPLANATION

Convert minutes to hours.

40 mins = $\frac{40}{60}$ h = $\frac{2}{3}$ h

10 mins = $\frac{10}{60}$ h = $\frac{1}{6}$ h

Tabulate given information.

Speed	Time	Distance
1u	$\frac{2}{3}$	
1u + 30	$\frac{1}{6}$	

Distance = Speed x Time

Speed	Time	Distance
1u	$\frac{2}{3}$	$\frac{2}{3}$u
1u + 30	$\frac{1}{6}$	$\frac{1}{6}$u + 5

↓

We know that Samuel drove a total distance of 85 km. ($\frac{2}{3}$u + $\frac{1}{6}$u + 5) → 85

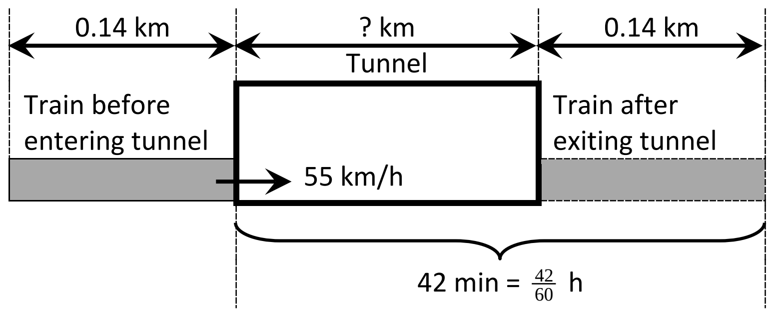
Work the equation. $\frac{5}{6}$u → 80

a) 1u → (80 km/h x $\frac{6}{5}$) = 96 km/h
b) (96 + 30) km/h = 126 km/h

a) Samuel's driving speed for the first 40 minutes was <u>96 km/h</u>.
b) Samuel's driving speed for the last 10 minutes was <u>126 km/h</u>.

3. A train that is 140 m long travels through a tunnel at a constant speed of 55 km/h. What is the length of the tunnel if the entire train took 42 minutes to pass through it?

WORKING



Distance = (55 km/h x $\frac{42}{60}$ h) = 38.5 km

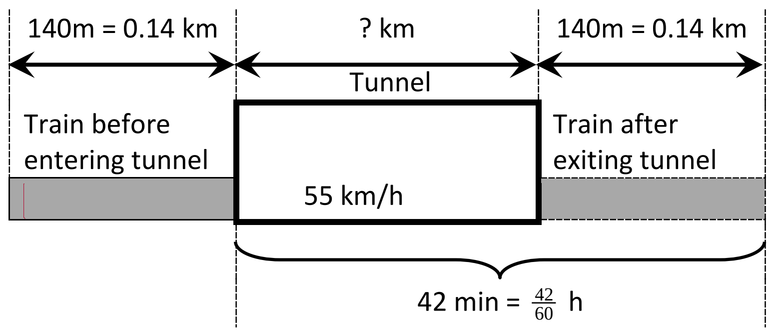
(38.5 – 0.14) km = 38.36 km

The length of the tunnel is 38.36 km.

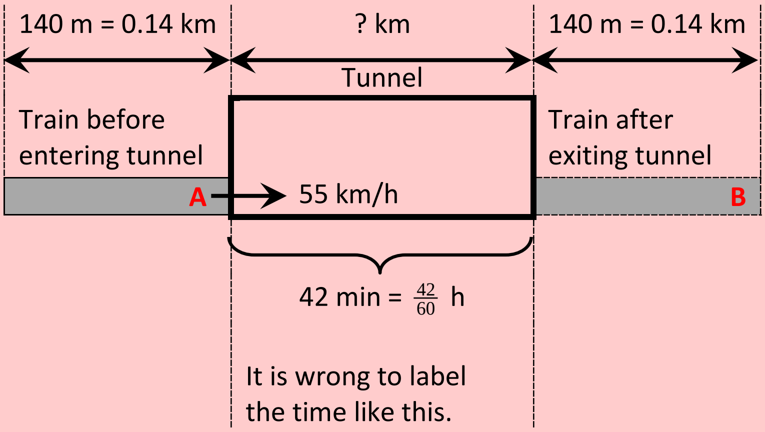
EXPLANATION

This example uses the **Logical Reasoning** method instead of the Unit Transfer Method in Speed and Rate. Diagram helps to put all the information into perspective.

Draw a diagram, complete with all given information. Convert all measurements into km and h.



CONFUSION ALERT

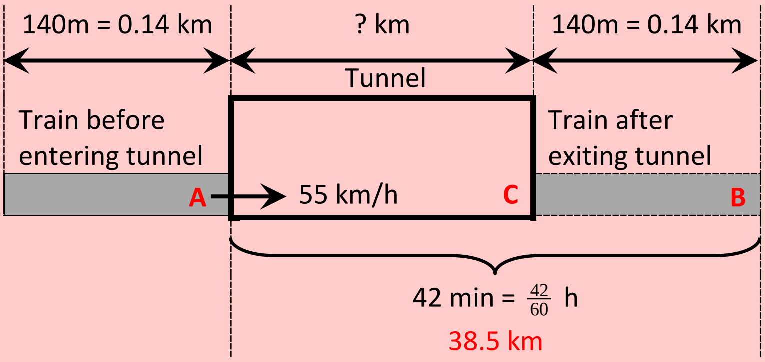


This is because the train completely travels through the tunnel only when its “head” moves from Point A to Point B.

Distance = Speed x Time = (55 km/h x $\frac{42}{60}$ h) = 38.5 km

CONFUSION ALERT

This is not the answer. The tunnel is not 38.5 km.
This is because we have just calculated the distance between A and B which includes tunnel length and train length.
What we want is the distance between A and C.
Therefore, we must deduct train length from the 38.5 km.



Deduct train length from 38.5 km.
(38.5 – 0.14) km = 38.36 km

The length of the tunnel is <u>38.36 km</u>.

4. John cycles from home to school every morning. He normally cycles to school at 6 m/s, and back home at 10 m/s. What is his average cycling speed for the whole trip?

WORKING

Assume distance = 60 m.
Time from Home to School: (60 m ÷ 6 m/s) = 10 s
Time from School to Home: (60 m ÷ 10 m/s) = 6 s
Average speed = (120 m ÷ 16 s) = 7.5 m/s

John's average cycling speed for the whole trip is <u>7.5 m/s</u>.

EXPLANATION

This example uses the <u>**Supposition**</u> method.

We are asked to calculate average speed.
Average speed = Total distance ÷ Total Time
Unfortunately, we don't know the total distance and the total time.

However, we know
the speed from Home to School, and
the speed from School to Home.

If we knew the distance, we could calculate
the time from Home to School, and
the time from School to Home.

Hence, we take a random figure to be the distance between Home and School.
Assume distance = 60 m

CONFUSION ALERT
Time = Distance ÷ Speed
We know the speeds (6 m/s and 10 m/s).

For the chosen figure for distance, we would want it to be
a common multiple of the figures of the two known speeds (6 and 10).
This is because later on, when we divide the common multiple by
6 m/s and 10 m/s, we will end up with whole numbers.
Whole numbers are easier to manage compared to fractions and decimals.

The quickest way to pick a common multiple of two numbers is
to multiply both numbers (6 x 10 = 60).
That is how we arrived at 60 m as the chosen figure for distance.

With this, we are able to calculate time.
Time = Distance ÷ Speed
Time from Home to School: (60 m ÷ 6 m/s) = 10 s
Time from School to Home: (60 m ÷ 10 m/s) = 6 s

Average speed from Home to School and from School to Home
= Total Distance ÷ Total Time
= (60 + 60) m ÷ (10 + 6) s
= 7.5 m/s

John's average cycling speed for the whole trip is 7.5 m/s.

CONFUSION ALERT

How is it that we are able to get the correct answer based on a random figure for distance between Home and School? We illustrate here.

Assume distance = (d) m

Total distance from Home to School and from School to Home
= (d + d) m = (2d) m

Time from Home to School: $(\frac{d}{6})$ s

Time from School to Home: $(\frac{d}{10})$ s

Total time from Home to School and from School to Home
$= (\frac{d}{6} + \frac{d}{10})$ s $= (\frac{10d}{60} + \frac{6d}{60})$ s $= (\frac{16d}{60})$ s

Average speed from Home to School and from School to Home
= Total distance ÷ Total time
= (2d) m ÷ $(\frac{16d}{60})$ s = (2d x $\frac{60}{16d}$) m/s = $\frac{60}{8}$ m/s = 7.5 m/s

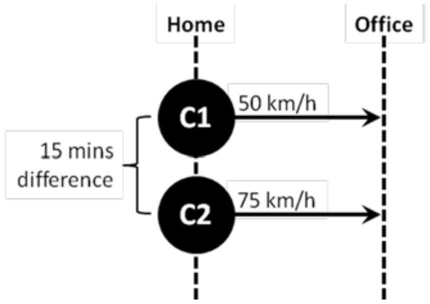
As you can see, the assumed figure will always be cancelled out.
So, no matter what the distance is assumed to be,
we will arrive at the same answer of 7.5 m/s.

CONFUSION ALERT

The use of a random figure for distance is only applicable in situations of average speed. This is because such situations are independent of the distance. This independence was seen in the cancelling out of the assumed distance (d) in the above Confusion Alert.

5. **Mr Chan drives from home to office at an average speed of 50 km/h daily. However, he woke up late one morning and left home 15 minutes later than usual. As a result, Mr Chan increased his driving speed by 25 km/h so that he could reach the office at his usual time. Find the distance between his home and office.**

WORKING



Same distance.

	Case 1	Case 2
Speed	50 2u	75 3u
Time	3p	2p

1p → 15 mins = $\frac{1}{4}$ h

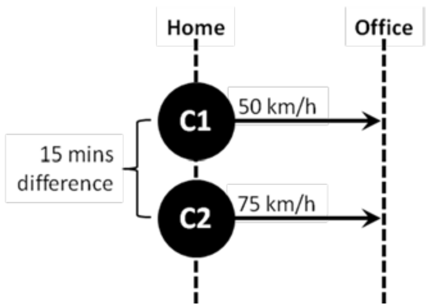
3p → (3 x $\frac{1}{4}$ h) = $\frac{3}{4}$ h

Distance = (50 km/h x $\frac{3}{4}$ h) = 37.5 km

The distance between Mr Chan's home and office is <u>37.5 km</u>.

EXPLANATION

This example uses the **Unit Transfer Method for Speed Problems**.



What we know:
- **Distance same** for both situations.

Hence, this is the rule to apply:
- When one object in two different situations travel the **same distance** (with known speeds), the **ratio of speed INVERTED** is the **ratio of time taken**.

	Case 1	Case 2
Speed	50 2u	75 3u
Time	3p	2p

← Convert to units.

← Invert speed ratio, but change units to parts.

CONFUSION ALERT:
We must invert only the ratio, not the absolute figures.
Hence, it is important to change units to parts.

In Case 2, Mr Chan left home 15 minutes late but arrived at the office at his usual time. Hence, 3p is 15 minutes more than 2p.

(3p – 2p) → 15 mins

1p → 15 mins = $\frac{1}{4}$ h

Now, focus on Case 1.

3p → (3 x $\frac{1}{4}$ h) = $\frac{3}{4}$ h

Distance = Speed x Time = (50 km/h x $\frac{3}{4}$ h) = 37.5 km

ALTERNATIVE:
We can also focus on Case 2 instead.

2p → (2 x $\frac{1}{4}$ h) = $\frac{2}{4}$ h = $\frac{1}{2}$ h

Distance = Speed x Time = (75 km/h x $\frac{1}{2}$ h) = 37.5 km

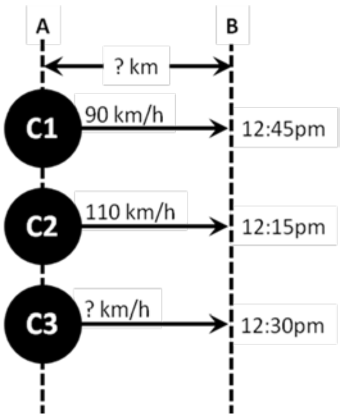
The distance between Mr Chan's home and office is 37.5 km.

6. **Richard is driving from Town A to Town B. If he drives at an average speed of 90 km/h, he will reach Town B at 12:45 pm. If he increases his driving speed by 20 km/h, he will reach Town B at 12:15 pm.**

a) Find the distance between Town A and Town B

b) Find the speed he should drive at to reach Town B at 12:30pm?

WORKING



Same Distance.

	Case 1	Case 2
Speed	90 9u	110 11u
Time	11p	9p

a) 2p → 30 mins

1p → (30 mins ÷ 2) = 15 mins

11p → (11 x 15 mins) = 165 mins = 2.75 h

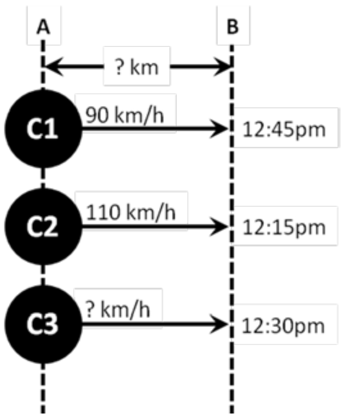
Distance = (90 km/h x 2.75 h) = 247.5 km

b) Speed = (247.5 km ÷ 2.5 h) = 99 km/h

a) The distance between Town A and Town B is <u>247.5 km</u>.

b) Richard should drive at a speed of <u>99 km/h</u> to reach Town B at 12:30pm.

EXPLANATION



What we know:

- **<u>Distance same</u>** for all three situations.

Hence, this is the rule to apply:

- When one object in two different situations travel the **<u>same distance</u>** (with known speeds), the **<u>ratio of speed INVERTED</u>** is the **<u>ratio of time taken</u>**.

	Case 1	Case 2
Speed	90 9u	110 11u
Time	11p	9p

← Convert to units in simplest form.

← Invert speed ratio, but change units to parts.

CONFUSION ALERT:
We must invert only the ratio, not the absolute figures.
Hence, it is important to change units to parts.

a) We know that the difference in time taken between the Case 1 and Case 2 is (11p – 9p) and 12:45pm – 12:15pm.
(11p – 9p) → 12:45pm – 12:15pm
2p → 30 mins
1p → (30 mins ÷ 2) = 15 mins

Now, focus on Case 1.
11p → (11 x 15 mins) = 165 mins = 2.75 h
Distance = Speed x Time = (90 km/h x 2.75 h) = 247.5 km

ALTERNATIVE:
We can also focus on Case 2 instead.
9p → (9 x 15 mins) = 135 mins = 2.25 h
Distance = Speed x Time = (110 km/h x 2.25 h) = 247.5 km

b) In Case 1, Richard arrives at 12:45pm.
In Case 3. Richard arrives at 12:30pm, which is 15 mins or 0.25 h earlier.

In Case 1, time = 2.75 h
In Case 3, time = (2.75 – 0.25) h = 2.5 h

ALTERNATIVE:
We can also focus on Case 2 instead.
In Case 2, Richard arrives at 12:15pm.
In Case 3, Richard arrives at 12:30pm, which is 15 mins or 0.25 h later.

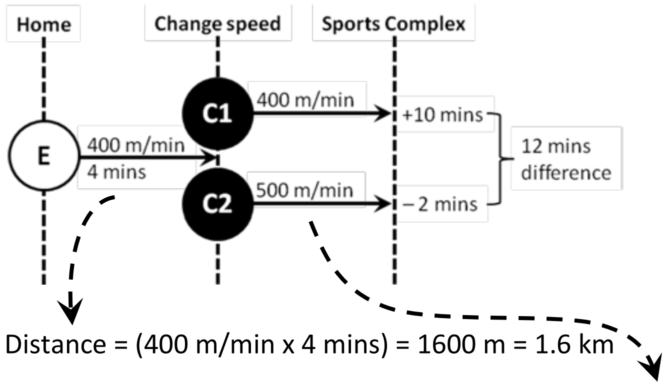
In Case 2, time = 2.25 h
In Case 3, time = (2.25 + 0.25) h = 2.5 h

Speed = Distance ÷ Time = (247.5 km ÷ 2.5 h) = 99 km/h

a) The distance between Town A and Town B is <u>247.5 km</u>.
b) Richard should drive at a speed of <u>99 km/h</u> to reach Town B at 12:30pm.

6. **Last Tuesday morning, Edward cycled from home to the sports complex for his swimming lesson. For the first 4 minutes, he cycled at an average speed of 400 m/min. When he realised that he was going to be late by 10 minutes, he quickly increased his speed by 100 m/min. As a result, he was early by 2 minutes. Find the distance between his home and the sports complex.**

WORKING



Distance = (400 m/min x 4 mins) = 1600 m = 1.6 km

Same distance.

	Case 1	Case 2
Speed	400 4u	500 5u
Time	5p	4p

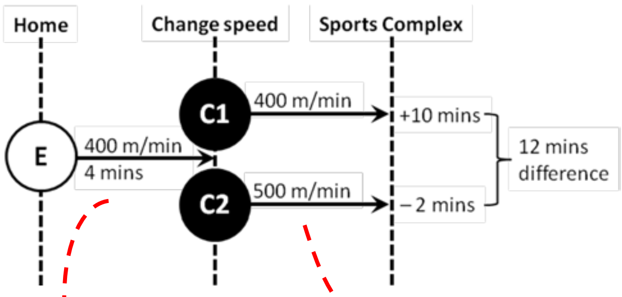
1p → 12 mins
5p → (5 x 12 mins) = 60 mins
Distance = (400 m/min x 60 mins) = 24000 m = 24 km

Total distance = (1.6 + 24) km = 25.6 km.

The distance between Edward’s home and the sports complex is 25.6 km.

EXPLANATION

This example uses the **Unit Transfer Method for Speed Problems**.



The first 4 minutes:
Distance = (400 m/min x 4 mins) = 1600 m = 1.6 km

The next segment:
There are two different situations (Case 1 and Case 2).

What we know:
- **Distance same** for both situations.

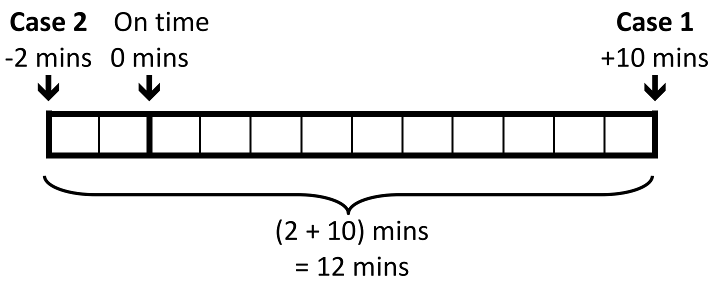
Hence, this is the rule to apply:
- When one object in two different situations travel the **same distance** (with known speeds), the **ratio of speed INVERTED** is the **ratio of time taken**.

	Case 1	Case 2
Speed	400 4u	500 5u
Time	5p	4p

← Convert to units in the simplest form.
← Invert speed ratio, but change units to parts.

CONFUSION ALERT:
We must invert only the ratio, not the absolute figures.
Hence, it is important to change units to parts.

In Case 1, Edward would have been 10 minutes late.
This means extra time taken, hence expected time + 10 mins.
In Case 2, Edward ended up 2 minutes early.
This means less time taken, hence expected time -2 mins.
Time difference between Case 1 and Case 2 = (10 + 2) mins = 12 mins.



CONFUSION ALERT:
Many students will work this step as
(10 – 2) mins = 8 mins which is wrong.
In number sentence format, it should be
(10 – (–2)) mins = (10 + 2) mins = 12 mins
However, students have not learnt about
negative numbers. Hence, it is best
to use diagram to work out this step.

We know that the difference in time taken
between Case 1 and Case 2 is (5p – 4p) and 12 minutes.
(5p – 4p) → 12 mins
1p → 12 mins

Now, focus on Case 1.
5p → (5 x 12 mins) = 60 mins
Distance = (400 m/min x 60 mins) = 24000 m = 24 km

ALTERNATIVE:
We can also focus on Case 2 instead.
4p → (4 x 12 mins) = 48 mins
Distance = (500 m/min x 48 mins) = 24000 m = 24 km

Total distance
= Distance in the first 4 minutes + Distance in the next segment
= (1.6 + 24) km = 25.6 km

The distance between Edward's home and the sports complex is <u>25.6 km</u>.

LET'S APPLY Problems Involving Speed: One Object Moving

1. Mr Sim drives his wife to work at 0800 every morning. If he drives at an average speed of 70 km/h, his wife will be 15 minutes late for work. However, if he increases his driving speed by 20 km/h, his wife will be 5 minutes early. Find Mr Sim's driving speed if his wife is to arrive for work on time.

2. Michelle drives to her office daily at 8:00am and at an average speed of 80 km/h. If she increases her speed by 10km/h, she will reach her office 5 minutes early. At what speed must she drive in order to reach her office at 8:30am?

3. Jennifer walked from her home to the shopping centre at an average speed of 4 km/h. After shopping, she took a bus home. The driver drove at an average speed of 36 km/h. Her trip home was 40 minutes less than her trip to the shopping centre. Find the distance between her home and the shopping centre.

4. If Mr Soh drives at an average speed of 80 km/h, he will reach the airport at 1:00pm. If he increases his driving speed by 20 km/h, he will reach the airport at 12:45pm. What should his average driving speed be if he wants to reach the airport at 12:30pm?

5. Alan walked from his home to the tuition centre. For the first 4 minutes, he walked at an average speed of 60 m/min. When he realised that he was going to be late by 15 minutes, he increased his speed to 75 m/min. However, he was still late by 5 minutes. Find the distance between his home and the tuition centre.

6. Mr Tham drove at an average speed of 90 km/h from home to school to fetch his son. Without spending any time at school, he drove his son back home along the same route at an average speed of 60 km/h. He took a total of 50 minutes for the whole journey. Find the distance between home and school

*7. Philip drives at a constant speed from City A to City B. If he increases his speed by 18 km/h, the time required will be reduced by 20%. However, if he reduces his speed by 12 km/h, he will take 55 minutes more to arrive at City B. Find the distance between City A and City B.

8. Adrian drives at an average speed of 40 km/h from his house to the office. On a particular day, after driving a distance of 10 km, he realised that his watch was slow. He increased his driving speed and managed to reach office on time. Later, he calculated and discovered that if he had driven at the increased speed right from the beginning of the journey, he would have reached the office 10 minutes earlier. What was his increased driving speed?

   (Hint: Use Model Method or Logical Reasoning Method)

9. Mr Lee left Town A for Town B at 9:00am. He travelled at an average speed of 60 km/h. After travelling 50 km, he increased his speed by 40 km/h to complete the remaining $\frac{5}{7}$ of his journey.
   a) At what time did he reach Town B?
   b) If he had travelled at 60 km/h for the whole journey, how many minutes later would he reach Town B?

   (Hint: Use Model Method or Logical Reasoning Method)

10. Mr Lim drove at a speed of 72 km/h from his office to the gym. After driving for 10 minutes, he realised that he had left his wallet in the office. He drove back to get it. At 1830, he reached his office. After spending 15 minutes looking for his wallet, he found it and continued driving to the gym. At 1905, he reached the gym. He did not change his driving speed throughout the journey. What was the total distance Mr Lim drove?

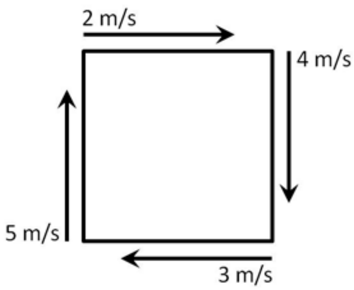
(Hint: Use Model Method or Logical Reasoning Method)

11. A train passes through Tunnel A and Tunnel B. Travelling at 12 m/s, the entire train takes 310 seconds to completely pass through Tunnel A, which is 3500 m long. At the same speed, the train takes 450 seconds to completely pass through Tunnel B. What is the length of Tunnel B?

(Hint: Use Model Method or Logical Reasoning Method)

12. Robert is jogging along a square track. The speeds at which he jogs along each of the four sides are 2 m/s, 4 m/s, 3 m/s and 5 m/s as shown in the figure below. What is Robert's average speed for the whole journey? (Round off your answer to 2 decimal places)

(Hint: Use Supposition Method)

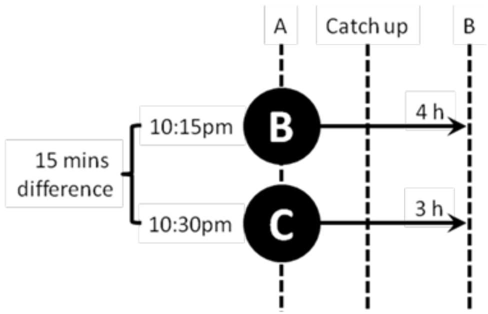


CHAPTER 2 SPEED: TWO OBJECTS MOVING IN THE SAME DIRECTION

EXAMPLES

1. Benson and Calvin drove from Town A to Town B. Benson left Town A at 10:15pm and took 4 hours to reach Town B. Calvin left 15 minutes later than Benson and took 3 hours to reach Town B. At what time did Calvin catch up with Benson?

WORKING



Same distance.

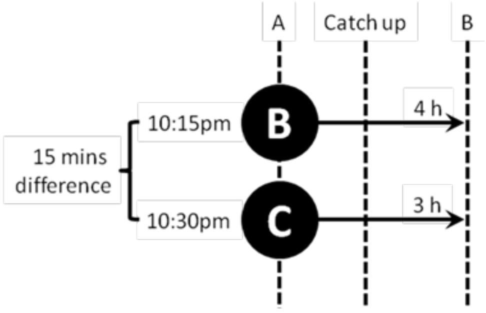
	B	C
Time	4u	3u
Speed	3p	4p

1u → 15 mins
3u → (3 x 15 mins) = 45 mins

10:30pm + 45 mins = 11:15pm

Calvin caught up with Benson at 11:15pm.

EXPLANATION



Focus on the "Catch up" point.

What we know:
- **Distance same** for the both persons.

Hence, this is the rule to apply:
- When two objects travel the **same distance** (with known time), the **ratio of time taken INVERTED** is the **ratio of speed**.

	B	C
Time	4 4u	3 3u
Speed	3p	4p

← Convert to units in simplest form.
← Invert time ratio, but change units to parts.

CONFUSION ALERT:
We must invert only the ratio, not the absolute figures.
Hence, it is important to change units to parts.

We know that Benson took 15 mins more than Calvin to reach the "Catch up" point.
(4u – 3u) → 15 mins
1u → 15 mins

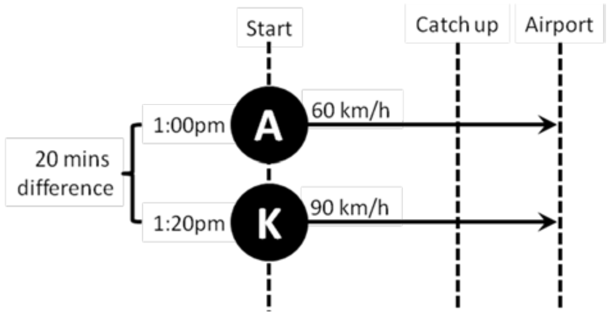
Now, focus on Calvin.
3u → (3 x 15 mins) = 45 mins
Time at which Calvin reaches the "Catch-up" point = 10:30pm + 45 mins = 11:15pm

ALTERNATIVE:
We can also focus on Benson instead.
4u → (4 x 15 mins) = 60 mins
Time at which Benson reaches the "Catch-up" point = 10:15pm + 60 mins = 11:15pm

Calvin caught up with Benson at 11:15pm.

2. **At 1:00pm, Alice drove at an average speed of 60 km/h towards the airport. 20 minutes later, Kevin drove at an average speed of 90 km/h towards the airport, from the same place Alice started her journey. When did Kevin catch up with Alice?**

WORKING



Same distance.

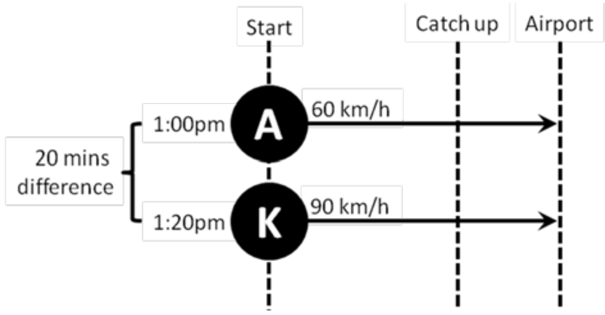
	A	K
Speed	60 2u	90 3u
Time	3p	2p

1p → 20 mins
2p → (2 x 20 mins) = 40 mins

1:20pm + 40 mins = 2:00pm

Kevin caught up with Alice at 2:00pm.

EXPLANATION



Focus on the “Catch-up” point.

What we know:

- **Distance same** for the both persons.

Hence, this is the rule to apply:

- When two objects
  travel the **same distance** (with known speed),
  the **ratio of speed INVERTED** is the **ratio of time**.

	A	K
Speed	60 2u	90 3u
Time	3p	2p

← Convert to units in simplest form.

← Invert time ratio, but change units to parts.

CONFUSION ALERT:
We must invert only the ratio, not the absolute figures.
Hence, it is important to change units to parts.

We know that Kevin left 20 minutes later than Alice.
(3p – 2p) → 20 mins
1p → 20 mins

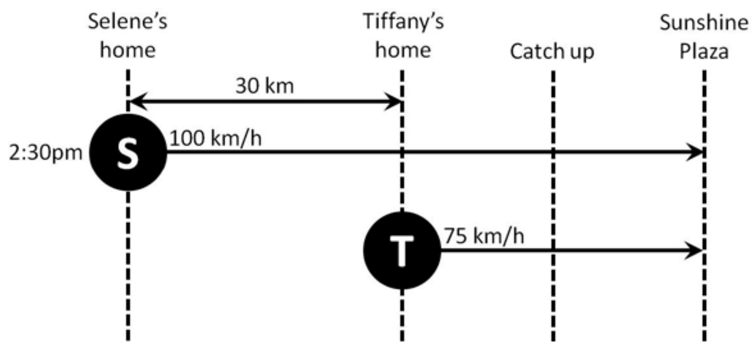
Now, focus on Kevin.
2p → (2 x 20 mins) = 40 mins
Time at which Kevin reaches the “Catch up” point = 1:20pm + 40 mins = 2:00pm

ALTERNATIVE:
We can also focus on Alice instead.
3p → (3 x 20 mins) = 60 mins
Time at which Alice reaches the “Catch up” point = 1:00pm + 60 mins = 2:00pm

Kevin caught up with Alice at 2:00pm.

3. Selene and Tiffany arranged to meet at Sunshine Plaza. At 2:30pm, they both left their houses and drove in the same direction towards the plaza. Selene's home was further away from the plaza. Selene and Tiffany drove at an average speed of 100 km/h and 75 km/h respectively. The distance between their houses was 30km. What time did Selene overtake Tiffany?

WORKING



Same time.

	S	T
Speed	100 4u	75 3u
Distance	4p	3p

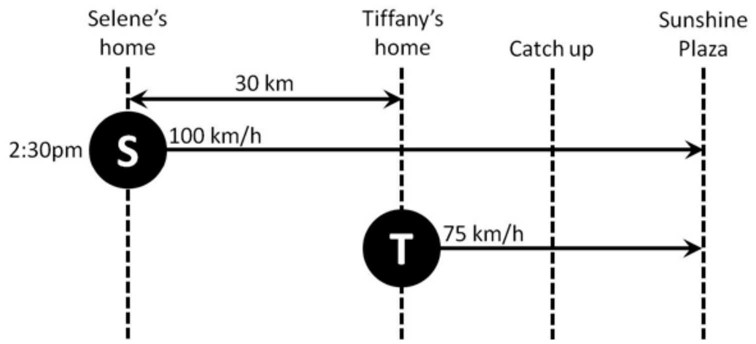
1p → 30 km
4p → (4 x 30 km) = 120 km

Time = (120 km ÷ 100 km/h) = 1 h 12 mins

2:30pm + 1 h 12 mins = 3:42pm

Selene caught up with Tiffany at <u>3:42pm</u>.

EXPLANATION



Focus on the "Catch up" point.

What we know:

- **<u>Time same</u>** for the both persons.

Hence, this is the rule to apply:

- When two objects
  travel the **<u>same time</u>** (with known distance),
  the **<u>ratio of distance</u>** is the **<u>ratio of speed</u>**.

	S	T
Speed	100 4u	75 3u
Distance	4p	3p

🡐 Convert to units in simplest form.
🡐 Keep speed ratio, but change units to parts.

CONFUSION ALERT:
We must keep only the ratio, not the absolute figures.
Hence, it is important to change units to parts.

We know that the difference between Selene's house to the "Catch up" point and Tiffany's house to the "Catch up" point is 30km.
(4p – 3p) → 30 km
1p → 30 km

Now, focus on Selene.
4p → (4 x 30) km = 120 km
Time = Distance ÷ Speed = (120 km ÷ 100 km/h) = 1 h 12 mins

ALTERNATIVE:
We can also focus on Tiffany instead.
3p → (3 x 30 km) = 90 km
Time = Distance ÷ Speed = (90 km ÷ 75 km/h) = 1 h 12 mins

2:30pm + 1 h 12 mins = 3:42pm

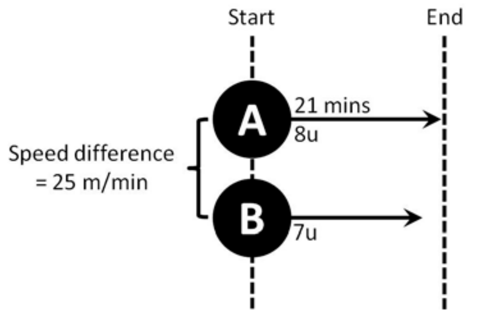
Selene caught up with Tiffany at <u>3:42pm</u>.

4. **Alex and Ben ran together in a race. Alex ran at 25 m/min faster than Ben. Alex completed the race in 21 minutes while Ben only managed to complete $\frac{7}{8}$ of the race.**

a) Find Ben's average running speed.

b) Find the time Ben took to complete the entire race.

WORKING



Same time.

	A	B
Distance	8u	7u
Speed	8p	7p

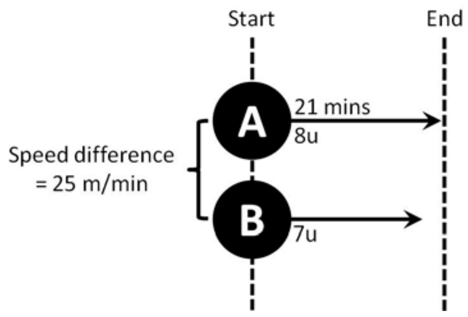
a) 1p → 25 m/min
7p → (7 x 25 m/min) = 175 m/min

b) 8p → (8 x 25 m/min) = 200 m/min
Distance = (200 m/min x 21 mins) = 4200 m

Time = (4200 m ÷ 175 m/min) = 24 mins

a) Ben's average jogging speed was <u>175 m/min</u>.
b) Ben took <u>24 mins</u> to complete the entire race.

EXPLANATION



What we know:

- **<u>Time same</u>** for the both persons.

Hence, this is the rule to apply:

- When two objects
  travel the **<u>same time</u>** (with known distance),
  the **<u>ratio of distance</u>** is the **<u>ratio of speed</u>**.

	A	B
Distance	8u	7u
Speed	8p	7p

← Already in units

← Keep distance ratio,
but change units to parts.

CONFUSION ALERT:
We must keep only the ratio, not the absolute figures.
Hence, it is important to change units to parts.

a) Alex was faster than Ben by 25 m/min.
(8p – 7p) → 25 m/min
1p → 25 m/min

Now, focus on Ben.
7p → (7 x 25 m/min) = 175 m/min

b) We need to calculate the time Ben took to complete the race.
Time = Distance ÷ Speed
We know the speed (175 m/min), but not the distance.

Now, focus on Alex to calculate the distance of the entire race.
8p → (8 x 25 m/min) = 200 m/min
Distance = Speed x Time = (200 m/min x 21 min) = 4200 m

Now, refocus on Ben.
Time = Distance ÷ Speed = (4200 m ÷ 175 m/min) = 24 mins

a) Ben's average jogging speed was <u>175 m/min</u>.
b) Ben took <u>24 mins</u> to complete the entire race.

5. Tom and Linden drove at uniform speed from City X to City Y. They started their journey at the same time. When Tom was $\frac{1}{3}$ the distance between City X and City Y, Linden was behind him by 8 km. Tom reached City Y 15 minutes before Linden. Given that Tom's speed was 116 km/h, find the distance between the two cities.

WORKING

Same time.

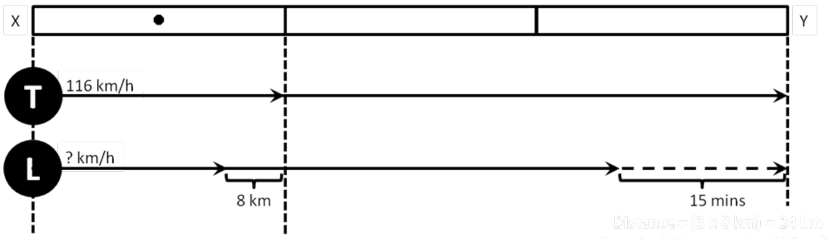
	T	L
Speed	116 29u	96 24u
Distance	29p	24p

5p → 8 km
1p → (8 km ÷ 5) = 1.6 km
29p → (29 x 1.6 km) = 46.4 km

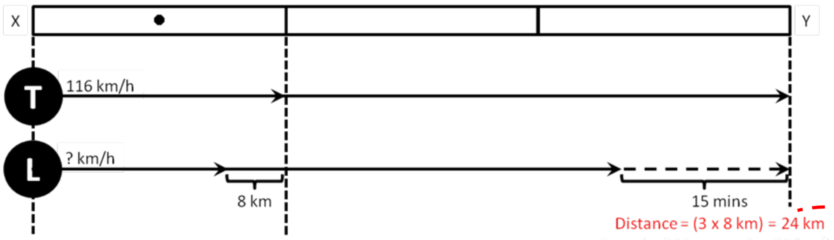
Distance = (46.4 km x 3) = 139.2 km

The distance between the two cities is 139.2 km.

EXPLANATION

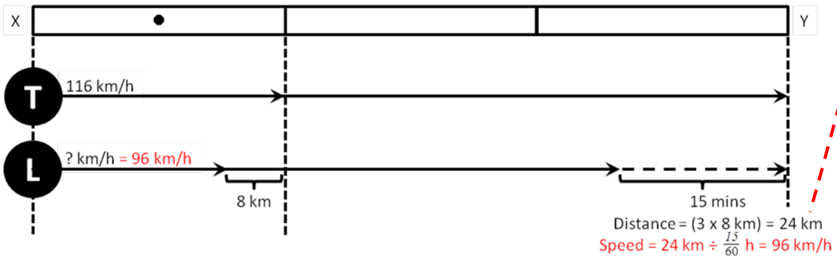


Draw a diagram, complete with all given information.



When Tom was $\frac{1}{3}$ into the journey, Linden trailed by 8 km.

Proportionately, when Tom completed the journey, Linden trailed by (3 x 8 km) = 24 km.



Speed
= Distance ÷ Time
= (24 km ÷ $\frac{15}{60}$ h)
= 96 km/h

What we know:

- **<u>Time same</u>** for both persons.

Hence, this is the rule to apply:

- When two objects travel the **<u>same time</u>** (with known speeds), the **<u>ratio of speed</u>** is the **<u>ratio of distance</u>**.

	T	L
Speed	116 29u	96 24u
Distance	29p	24p

🡐 Convert to units in simplest form.
🡐 Keep speed ratio, but change units to parts.

CONFUSION ALERT:
We must keep only the ratio, not the absolute figures.
Hence, it is important to change units to parts.

(29p – 24p) → 8 km
5p → 8 km
1p → (8 km ÷ 5) = 1.6 km

We know that Tom completed the journey.
So, Distance Tom travelled = Distance between both cities.

Now focus on Tom.
29p → (29 x 1.6 km) = 46.4 km
This is just $\frac{1}{3}$ of the distance Tom travelled.
Total distance Tom travelled = (3 x 46.4 km) = 139.2 km

The distance between the two cities is 139.2 km.

ALTERNATIVE #1:
We can also focus on Linden instead.
24p → (24 x 1.6 km) = 38.4 km
This is just $\frac{1}{3}$ of the distance Linden travelled
when Tom completed $\frac{1}{3}$ of the journey.
Distance Linden travelled when Tom completed the journey
= (3 x 38.4 km) = 115.2 km

Distance between the two cities
= (115.2 + Distance Linden has yet to travel) km
= (115.2 + 24) km
= 139.2 km

ALTERNATIVE #2:
We can straight away consider the entire journey (instead of $\frac{1}{3}$ the journey).
(29p – 24p) → 24 km
5p → 24 km
1p → (24 km ÷ 5) = 4.8 km

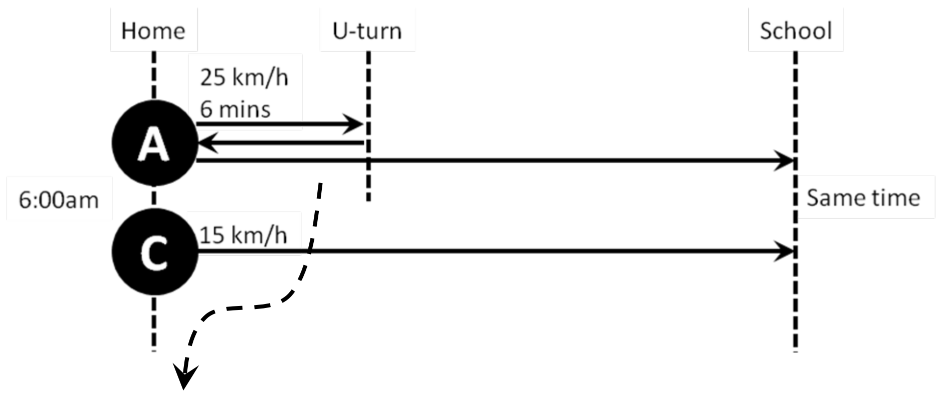
Now focus on Tom.
29p → (29 x 4.8 km) = 139.2 km

ALTERNATIVE #3:
Looking at Alternative #2, we can also focus on Linden instead.
24p → (24 x 4.8 km) = 115.2 km
This is the distance Linden travelled when Tom completed the journey.

Distance between the two cities
= (115.2 + Distance Linden still had to travel) km
= (115.2 + 24) km
= 139.2 km

6. Every morning at 6:00am, Alan cycles from home to school at an average speed of 25 km/h. His brother, Calvin cycles at 15 km/h. One particular day, after cycling for 6 minutes, Alan realised that he had forgotten to bring his wallet. He cycled back home immediately while Calvin continued on towards school. Alan reached school at the same time as Calvin. Find the distance between their home and school.

WORKING



(25 km/h x $\frac{6}{60}$ h) = 2.5 km

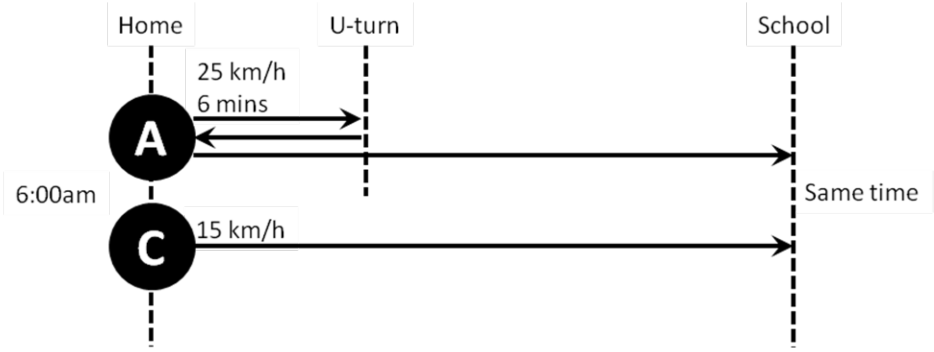
Same time.

	A	C
Speed	25 5u	15 3u
Distance	5p	3p

2p → (2 x 2.5 km) = 5 km
1p → (5 km ÷ 2) = 2.5 km
3p → (3 x 2.5 km) = 7.5 km

The distance between their home and school is 7.5 km.

EXPLANATION



The first 6 minutes:
Alan travelled at 25 km/h for 6 minutes.
He travelled a certain distance = Speed x Time = (25 km/h x $\frac{6}{60}$ h) = 2.5 km.

The entire journey:

What we know:
- **Time same** for both brothers.

Hence, this is the rule to apply:
- When two objects
  travel the **same time taken** (with known speeds),
  the **ratio of speed** is the **ratio of distance**.

	A	C
Speed	25 5u	15 3u
Distance	5p	3p

← Convert to units in simplest form.
← Keep speed ratio, but change units to parts.

CONFUSION ALERT:
We must keep only the ratio, not the absolute figures.
Hence, it is important to change units to parts.

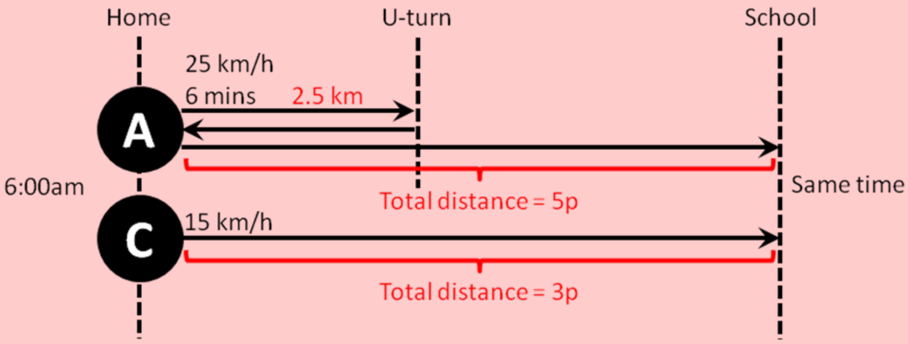
Look at the UTM table: The difference in distance travelled by the brothers is (5p – 3p).

Look at the diagram: The difference in distance travelled by the brothers is the part where Alan travelled towards school for 6 minutes (2.5 km) and back home again (2.5 km).

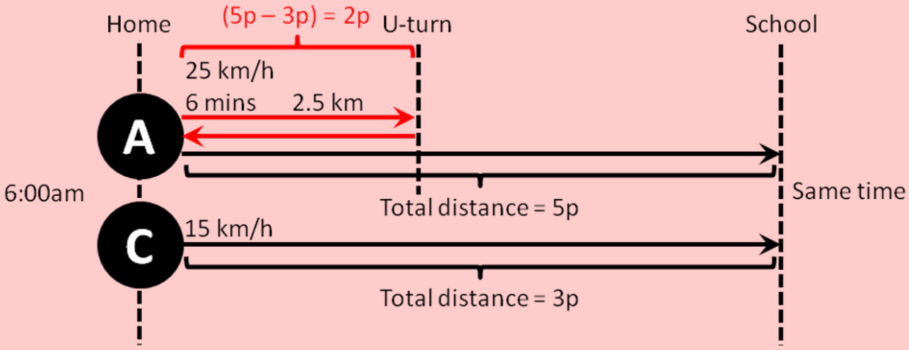
Equate these:
(5p – 3p) → (2 x 2.5 km)
2p → 5 km
1p → (5 km ÷ 2) = 2.5 km

ALTERNATIVE:

Label the diagram with the derived information.



Derive more information.



From the diagram, we can see that:
2p → (2 x 2.5 km) = 5 km
1p → (5 km ÷ 2) = 2.5 km

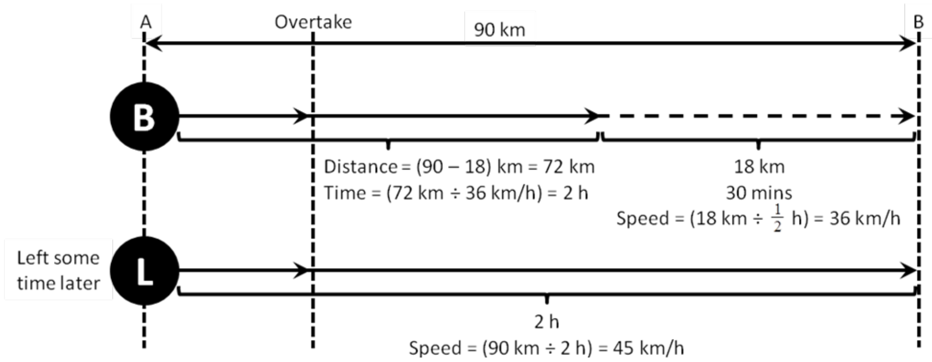
Now, focus on Calvin.
Calvin travelled a distance of 3p.
3p → (3 x 2.5 km) = 7.5 km

ALTERNATIVE:
We can also focus on Alan instead.
Alan travelled a distance of 5p.
Of this, we already determined that 2p is the extra distance travelled.
Hence, the part where Alan travelled directly from home to school is (5p – 2p) = 3p.
3p → (3 x 2.5 km) = 7.5 km

The distance between their home and school is 7.5 km.

7. **Town A and Town B are 90 km apart. Bala started out from Town A and drove towards Town B. Some time later, Leon left Town A and drove towards Town B along the same route. On the way, he overtook Bala and arrived at Town B 30 minutes earlier than Bala. When Leon arrived at Town B, Bala was still 18 km away from Town B. What is Leon's average driving speed?**

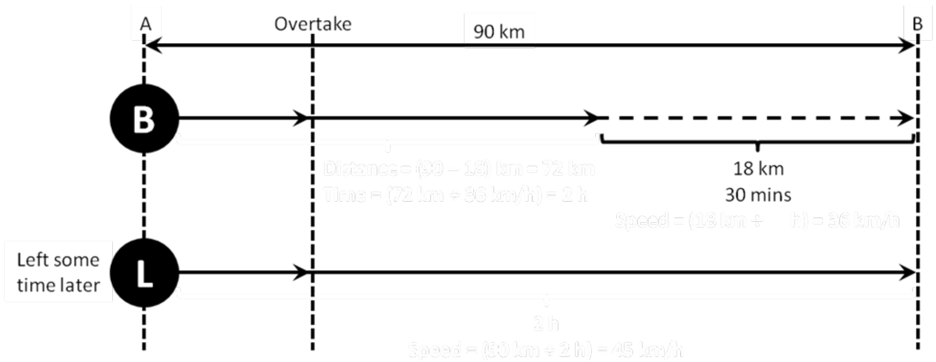
WORKING



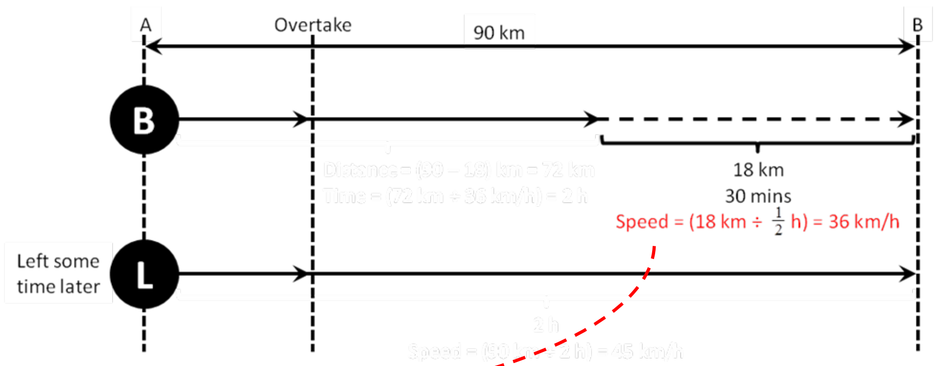
Leon's average driving speed was 45 km/h.

EXPLANATION

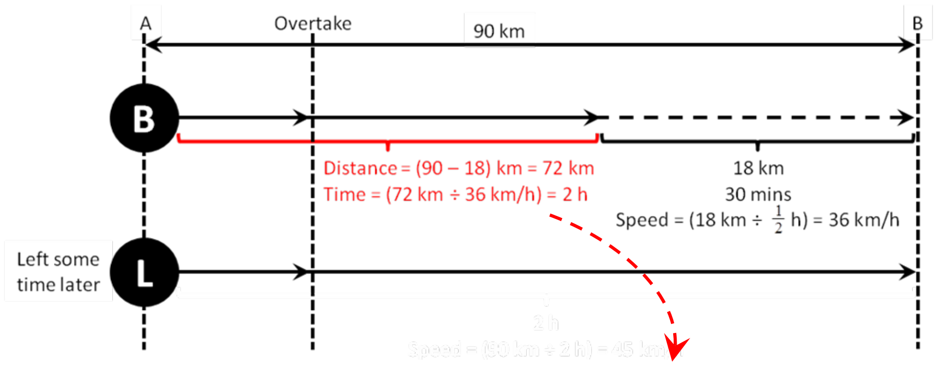
Draw a diagram, complete with all the given information.



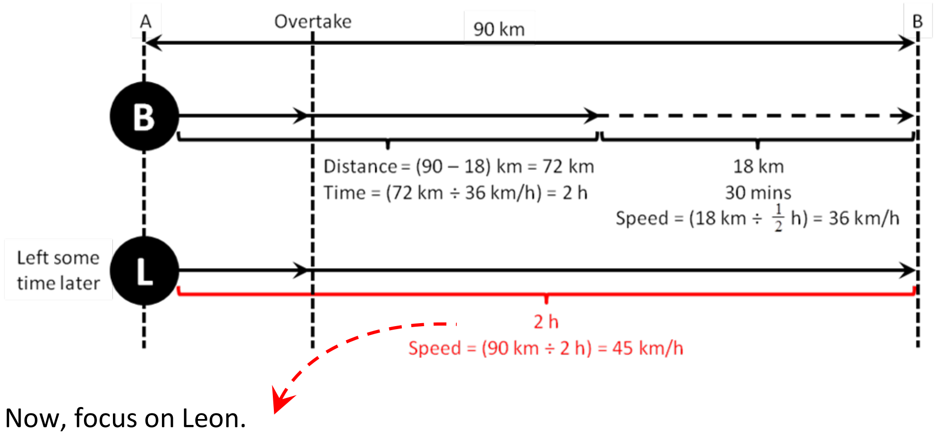
Derive further information.



Focus on Bala.
When Leon completed the journey, Bala was 18 km and 30 mins behind.
Speed = Distance ÷ Time = (18 km ÷ $\frac{30}{60}$ h) = 36 km/h



When Leon completed the journey, Bala had travelled (90 – 18) km = 72 km.
We had calculated that Bala's average speed was 36 km/h.
Time = Distance ÷ Speed = (72 km ÷ 36 km/h) = 2 h



Now, focus on Leon.
When Bala completed 72 km in 2 h, Leon completed 90 km also in 2 h.
Speed = Distance ÷ Time = (90 km ÷ 2 h) = 45 km/h

Leon's average driving speed was <u>45 km/h</u>.

LET'S APPLY Problems Involving Speed: Two Objects Moving in the Same Direction

1. A lorry driver left Town X at an average speed of 60 km/h towards Town Y. 45 minutes later, the driver of a car left Town X and travelled along the same route at an average speed of 90 km/h. The car overtook the lorry when they were 20% into the journey. Find the distance between the two towns.
(Hint: Involves Same distance)

2. At 4:00pm, a motorcyclist left Town A for Town B and travelled at a uniform speed. Two hours later, a car driver began his journey from Town A and travelled along the same route. The car driver overtook the motorcyclist at 7:30pm. The car driver travelled at 60 km/h faster than the motorcyclist.
   a) What was the speed of the motorcyclist?
   b) What was the distance between Town A and Town B if the car driver was 40km away from Town B at 7:30pm?
(Hint: Involves Same distance)

3. At 10:00am, Zen left City A and drove towards City B at a constant speed of 88 km/h. 15 minutes later, Tim left City A and drove towards City B at a constant speed of 99 km/h. Both of them reached City B at the same time.
   a) Find the distance between City A and City B.
   b) Find the time Tim took to reach City B.
(Hint: Involves Same distance)

4. Andy and Bernice started jogging from Park A to Park B at the same time. Andy jogged at an average speed of 225 m/min while Bernice jogged at an average speed of 135 m/min. After jogging for 30 minutes, Andy decided to turn back and jog towards Bernice. Andy met Bernice midway between Park A and Park B. Andy and Bernice continued their jog toward Park Y together. What was the distance between Park A and Park B?
(Hint: Involves Same time)

5. Samuel and Jimmy started out from Town X towards Town Y at the same time, at an average speed of 88 km/h and 60 km/h respectively. After driving for 15 minutes, Jimmy increased his average speed to 96 km/h. If Samuel and Jimmy reached Town Y at the same time, find the distance between Town X and Town Y.
(Hint: Involves Same time)

6. Car A, Car B and Car C were waiting along a straight road for the race to start. Car C was 3 km ahead of Car B and Car B was 1 km ahead of Car A. At 10:15am, the race started. Car A overtook Car B in 10 minutes. In another 5 minutes, Car A overtook Car C. If Car B's speed was 150 km/h, what was the time when Car B overtook Car C?
(Hint: Involves Same time)

7. Two cyclists, Alice and Karen were on a straight road waiting for the race to start. Alice was 1800 m ahead of Karen. At 11:00am, the race started. Alice cycled at 7 m/s while Karen cycled at 10 m/s. At what time did Karen catch up with Alice?
(Hint: Involves Same time)

8. Steven and Tom arranged to meet at a cinema for movie after work. At 6:00pm, both left their offices and drove in the same direction towards the cinema. Steven, whose office was further away from the cinema, drove at an average speed of 96 km/h. Tom drove at an average speed of 72 km/h. The distance between Steven's office and Tom's office was 20 km. At what time did Steven overtake Tom?

(Hint: Involves Same time)

9. Samuel drove at an average speed of 90 km/h from his office to a restaurant. 20 km from the restaurant, Samuel realised that he had left his wallet back at the office. He dropped his colleague, Calvin at a taxi-stand there and drove back to the office, maintaining his speed. Calvin continued on to the restaurant in a taxi which travelled at an average speed of 50 km/h. Both men reached the restaurant at the same time. What is the distance between the office and the restaurant?

(Hint: Involves Same time)

*10. Tom, Ben and Harry travelled from Point X to Point Y. They left Point X at the same time. Tom drove at a speed of 63 km/h, Ben jogged at 9 km/h and Harry walked at 7 km/h. Ben took a lift from Tom for the first part of his journey, then jogged the remaining 2.8 km. When Ben alighted, Tom drove back to pick Harry. All three arrived at Point Y at the same time. How far did Harry walk?

(Hint: Involve same time)

*11. Philip and Jason are cycling on a straight track. If Philip started 200m ahead of Jason, and both of them started cycling at the same time, Jason will take 50 seconds to overtake Philip. If both of them started from the same position, but Philip started cycling 20 seconds before Jason, Jason will take 40 seconds to catch up with Philip. Find the average cycling speed of each boy.

(Hint: Involves Same time, Same distance)

12. Tom and Leon drove from Town X to Town Y. Tom started his journey at 11:00am, travelling at an average speed of 72 km/h. Leon started his journey some time later. At 11:15am, Leon overtook Tom. When Leon reached Town Y at 12:25pm, Tom was still 21 km away from Town Y.

a) Find Leon's average speed.

b) At what time did Leon start his journey?

(Hint: Use Logical Reasoning Method)

13. Martin started jogging from Point X to Point Y at an average speed of 12 km/h. Some time later, Carl started jogging from Point X to Point Y. He overtook Martin after jogging $\frac{5}{9}$ of the distance. From there, Carl took 4.4 minutes to reach Point Y. Given that Martin was 3.12 km away when Carl reached Point Y, how long did Martin take to jog from Point X to Point Y?

(Hint: Use Logical Reasoning Method)

ANSWERS

Answers to questions in the prior chapters' Let's Apply sections are listed in this chapter. Detailed workings may be downloaded at:

www.mathsheuristics.com/?page_id=472

CHAPTER 1 SPEED: ONE OBJECT MOVING

LET'S APPLY Problems Involving Speed: One Object Moving

1. 84 km/h
2. 120 km/h
3. 3 km
4. $133\frac{1}{3}$ km/h
5. 3240 m
6. 30 km

*7. 330 km

8. 120 km/h
9. a) 11:05am
   b) 50 mins
10. 48 km
11. 5180 m
12. 3.12 m/s

CHAPTER 2 SPEED: TWO OBJECTS MOVING IN THE SAME DIRECTION

LET'S APPLY Problems Involving Speed: Two Objects Moving in the Same Direction

1. 675 km
2. a) 45 km/h
   b) 197.5 km
3. a) 198 km
   b) 12:15 pm
4. 10125 m
5. 99 km
6. 10:33 am
7. 11:10 am
8. 6:50 pm
9. 28 km

*10. 2.1 km

*11. Jason's speed is 12 m/s
Philip's speed is 8 m/s

12. a) 90 km/h
    b) 11:03am
13. 45 mins

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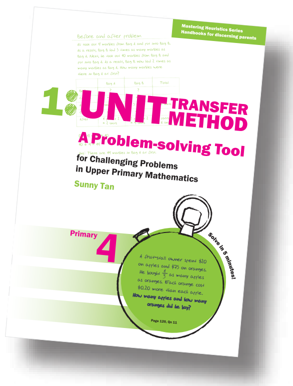

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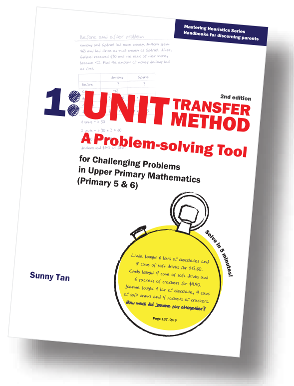

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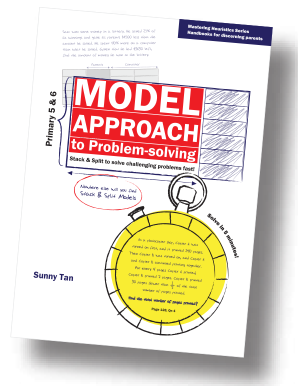

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Sunny Tan trains students, especially those in the PSLE year, in the use of various Heuristics techniques.
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