

LESSON 1: Systematic Listing

Working Systematically

Making lists and tables are two ways of working systematically. They help us to organize our thoughts in a logical and systematic way

(a) **Make a list**

When a organized list is used, we should arrange it in such a way that there is some natural **pattern**,

(b) **Make a Table**

Putting information gathered from the question in a table helps us to see the problem more clearly.

GUIDED EXAMPLE 1

When a certain number is divided by 3, the remainder is 1.

When this number is divided by 7, the remainder is 3.

[This number is between 60 and 90.] *Range*

What are the possible number(s)?

Example :

$$\square \div 3 = 4 \text{ R } 1$$

$$\underline{13} = 4 \times 3 + 1$$

| | |
|----|---|
| M3 | 57 , 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90 |
| +1 | 58 , 61, 64, 67, 70, (73), 76, 79, 82, 85, 88, 91 |
| M7 | 56 , 63, 70, 77, 84 |
| +3 | 59 , 66, (73), 80, 87 |

← possible original no.

Ans : 73

GUIDED EXAMPLE 2

Range

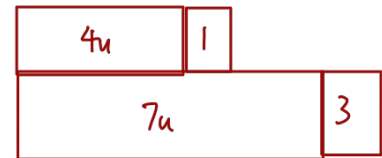
I have a certain number of rubber bands, between 50 and 90.
 If I put them into bags of 4, I will have 1 rubber band left behind.
 If I put them into bags of 7, I will have 3 rubber bands left behind.
 How many rubber bands do I have?

| | |
|----|---|
| M4 | 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88 |
| +1 | 49, 53, 57, 61, 65, 69, 73, 77, 81, 85, 89 |
| M7 | 49, 56, 63, 70, 77, 84 |
| +3 | 52, 59, 66, 73, 80, 87 |

Ans : 73

Excess & Shortage does not work

$$4u + 1 \neq 7u + 3$$



∴ the number of bags are different

GUIDED EXAMPLE 3

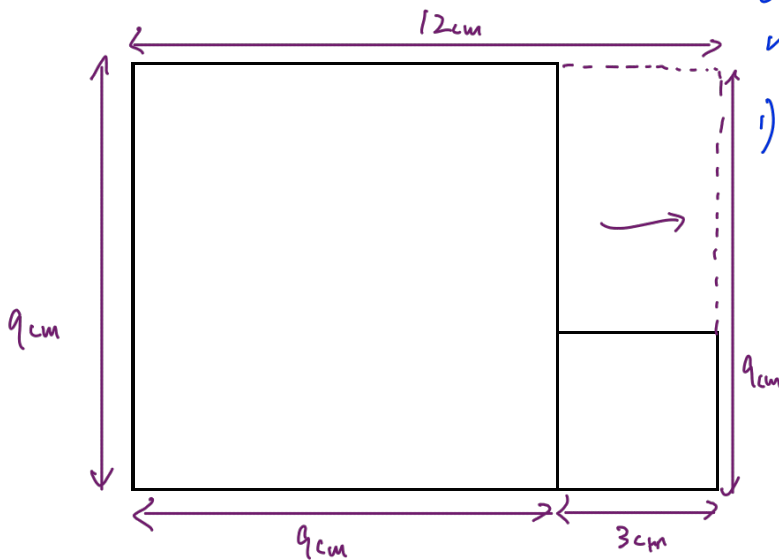
The figure below is not drawn to scale.

Two squares are joined and the total area of these two squares is 90 cm^2 .

Given that the sides of the squares are whole numbers, when measured in cm,

What is the perimeter of the figure?

Hint to use Listing



3 cm ✓ 5 cm ✓ 0.6 cm ✗ $3\frac{2}{3}$ cm ✗

1) List out lengths & areas of squares

| Length | x | Length | = | Area |
|---------------|--------------|---------------|--------------|----------------|
| 1 | x | 1 | = | 1 |
| 2 | x | 2 | = | 4 |
| 3 | x | 3 | = | 9 |
| 4 | x | 4 | = | 16 |
| 5 | x | 5 | = | 25 |
| 6 | x | 6 | = | 36 |
| 7 | x | 7 | = | 49 |
| 8 | x | 8 | = | 64 |
| 9 | x | 9 | = | 81 |
| 10 | x | 10 | = | 100 |

2) Look for 2 areas that add up to 90 cm^2 .
 $81 + 9 = 90$

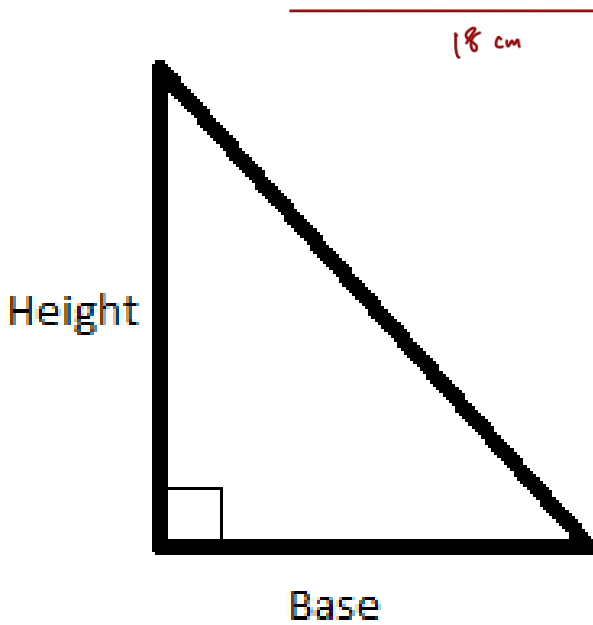
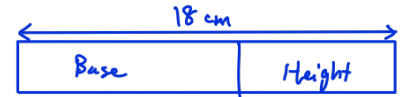
3) Lengths of squares are 3 cm & 9 cm

4) Perimeter of figure = $2(9 + 12)$
 = 42

Ans : 42 cm

GUIDED EXAMPLE 4

Karen drew some right – angled triangles like the one shown below.
 For every triangle, the base and the height sum up to 18 cm.
 The base and height are in whole numbers *when measured in cm.*
 What is the largest possible area that this triangle can cover?



| | | | | | |
|----------------------|--------------|----------|------------|-----|-------------|
| $\frac{1}{2} \times$ | \downarrow | \times | \uparrow | $=$ | Area |
| | Base | | Height | | |
| $\frac{1}{2} \times$ | 18 | \times | 0 | $=$ | 0 |
| $\frac{1}{2} \times$ | 17 | \times | 1 | $=$ | 8.5 |
| $\frac{1}{2} \times$ | 16 | \times | 2 | $=$ | 16 |
| $\frac{1}{2} \times$ | 15 | \times | 3 | $=$ | 22.5 |
| $\frac{1}{2} \times$ | 14 | \times | 4 | $=$ | 28 |
| $\frac{1}{2} \times$ | 13 | \times | 5 | $=$ | 32.5 |
| $\frac{1}{2} \times$ | 12 | \times | 6 | $=$ | 36 |
| $\frac{1}{2} \times$ | 11 | \times | 7 | $=$ | 38.5 |
| $\frac{1}{2} \times$ | 10 | \times | 8 | $=$ | 40 |
| $\frac{1}{2} \times$ | 9 | \times | 9 | $=$ | <u>40.5</u> |
| $\frac{1}{2} \times$ | 8 | \times | 10 | $=$ | 40 |

Ans : 40.5 cm²

GUIDED EXAMPLE 5

There are 3 light bulbs in a shop. ^{M2}

The red bulb flashes every 2 minutes.

The blue bulb flashes every 3 minutes. ^{M3}

The green bulb flashes every 8 minutes. ^{M8}

All the bulbs light up together when Henry walked into the shop.

How many times will he see at least 2 bulbs light up together if he was in the shop for 32 minutes? *Range*

| | |
|----|--|
| M2 | 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32 |
| M3 | 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 |
| M8 | 8, 16, 24, 32 |

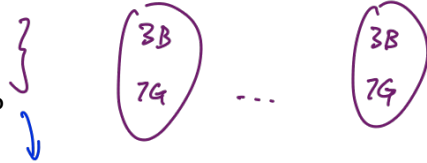
6, 8, 12, 16, 18, 24, 30, 32

Ans : 8 times

GUIDED EXAMPLE 6

Mr Lee wants to put all 36 boys and 90 girls into groups for an activity.
 There are more girls than boys in each group.
 Each group must have the same number of boys.
 Each group must also have the same number of girls.
 What is the greatest number of groups Mr Lee can form?

Example :



(Nan Hua P5 CA2)

Factor pair listing

* Highest Common factor

- Boys
 36
 = 1 x 36
 = 2 x 18
 = 3 x 12
 = 4 x 9
 = 6 x 6

- Girls
 90
 = 1 x 90
 = 2 x 45
 = 3 x 30
 = 5 x 18
 = 6 x 15
 = 9 x 10

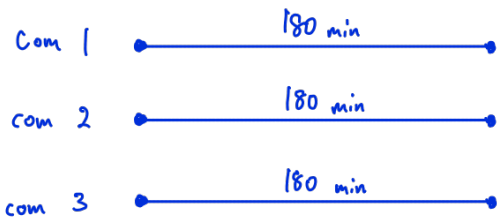
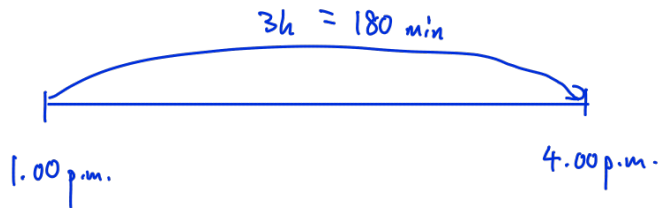
18 groups of 2 Boys & 5 girls

Ans : 18

GUIDED EXAMPLE 7

Similar to PSLE 2017 P2 Q5

5 friends shared 3 computers to play a game from 1:00 pm to 4:00 pm.
 They took turns to play on the 3 computers and all of them played for the same time.
 How many minutes did each person play?



$$\begin{aligned} \text{Total available} &= 3 \times 180 \text{ min} \\ \text{play time} &= 540 \text{ min} \\ \text{Time per} &= 540 \text{ min} \div 5 \\ \text{friend} &= 108 \text{ min} \end{aligned}$$

Ans : 108 min

BUILD YOUR UNDERSTANDING

1. The number of pupils participating in a race is from 32 and 60.
 If the competitors get into groups of 6, the last group will have 2 pupils.
 If they get into groups of 8, the last group will have 4 pupils.
 How many pupils are there in the race?

(Ai Tong P5 SA1 2012)

| | |
|----|----------------------|
| M6 | 30, 36, 42, 48, 54 |
| +2 | 32, 38, (44), 50, 56 |
| M8 | 32, 40, 48, 56 |
| +4 | 36, (44), 52, 60 |

Ans : 44

P5 Heuristics Approach to Problem Solving

Systematic Listing

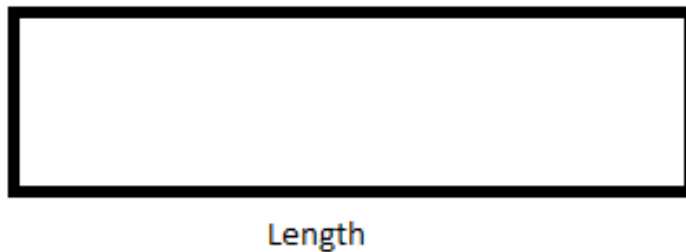
2. When a number is divided by 5, the remainder is 2.
 When the same number is divided by 6, the remainder is 1.
 If this number is between 20 to 50, what are the possible number(s)?

Range

| | |
|------|---|
| $M5$ | 15, 20, 25, 30, 35, 40, 45, 50 |
| +2 | 17, 22, 27, 32, <u>37</u> , 42, 47, 52 |
| $M6$ | 18, 24, 30, 36, 42, 48, 54 |
| +1 | 19, 25, 31, <u>37</u> , 43, 49, 55 |

Ans : 37

3. Kenny drew some rectangles like the one shown below.
 The perimeter of the rectangle at any one time is 32 cm.
 The breadth and length are whole numbers, *when measured in cm.*
 What is the largest possible area that such a rectangle can cover?



Length & Breadth can be the same.

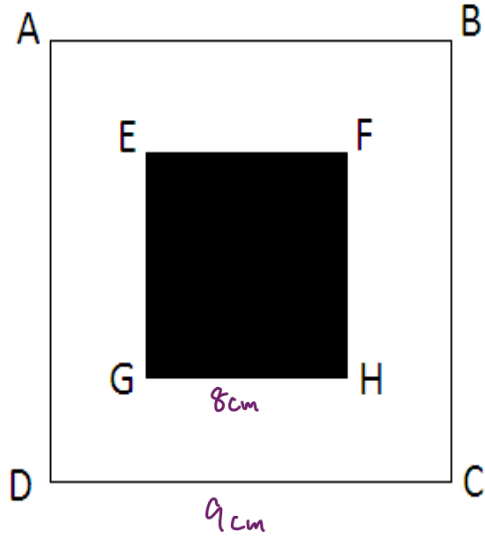
$$\begin{aligned} \text{Length} + \text{Breadth} &= 32 \div 2 \\ &= 16 \end{aligned}$$

| Length | x | Breadth | = | Area |
|--------|---|---------|---|------|
| 15 | x | 1 | = | 15 |
| 14 | x | 2 | = | 28 |
| 13 | x | 3 | = | 39 |
| 12 | x | 4 | = | 48 |
| 11 | x | 5 | = | 55 |
| 10 | x | 6 | = | 60 |
| 9 | x | 7 | = | 63 |
| 8 | x | 8 | = | 64 |

Ans: 64 cm²

4. The figure below shows 2 squares, ABCD and EFGH.
 The length of each square is a whole number, when measured in cm.
 The area of the unshaded part is 17 cm^2 .
 What is the perimeter of square EFGH?

Big square area - small square area



| Length | x | Length | = | Area |
|----------|---|--------|---|------|
| 1 | x | 1 | = | 1 |
| 2 | x | 2 | = | 4 |
| 3 | x | 3 | = | 9 |
| 4 | x | 4 | = | 16 |
| 5 | x | 5 | = | 25 |
| 6 | x | 6 | = | 36 |
| 7 | x | 7 | = | 49 |
| <u>8</u> | x | 8 | = | 64 |
| <u>9</u> | x | 9 | = | 81 |

$$81 - 64 = 17$$

$$4 \times 8 = 32$$

Ans : 32 cm

P5 Heuristics Approach to Problem Solving

Systematic Listing

5. In a music lesson, Belinda rings a bell once every 2 seconds while Alice claps once every 3 seconds. A timer starts at 8am and none of them clapped or rang the bell. How many times can a ring and a clap be heard together within 24s?

| | | |
|--------|-------|--|
| (Bell) | m_2 | 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 |
| (Clap) | m_3 | 3, 6, 9, 12, 15, 18, 21, 24 |

6, 12, 18, 24

Ans: 4 times.

P5 Heuristics Approach to Problem Solving

Systematic Listing

6. There are two neon lights in a shop.
The red light flashes every 6 minutes and the yellow light flashes every 9 minutes.
Both neon lights flashed together when Annette walked into the shop.

* Including the flashes Annette saw when she first stepped into the shop, *
how many times will she see the 2 neon lights flash together
if she stays in the shop for 1 hour? = 60 min

(Henry Park P6 SA2 Q13)

| | |
|----|---------------------------------------|
| m6 | 6, 12, 18, 24, 30, 36, 42, 48, 54, 60 |
| m9 | 9, 18, 27, 36, 45, 54 |

$$3 + 1 = 4$$

Ans: 4 times

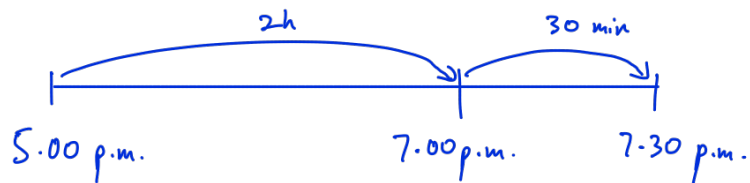
P5 Heuristics Approach to Problem Solving

Systematic Listing

7. 6 friends shared 5 roller blades to skate along East Coast Park.
Each of them skated for 125 minutes.
If they started skating at 5:00pm, what time will they end?

$$\begin{aligned} \text{Total available} &= 6 \times 125 \text{ min} \\ \text{play time} &= 750 \text{ min} \end{aligned}$$

$$\begin{aligned} \text{Time they} &= 750 \text{ min} \div 5 \\ \text{skated for} &= 150 \text{ min} \\ &= 2\text{h } 30\text{min} \end{aligned}$$



Ans : 7.30 p.m.