

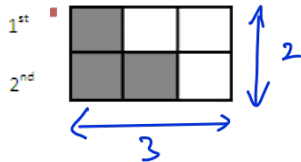
LESSON 3 Look for Pattern – Number Sequence

**GUIDED ACTIVITY
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Sum of Consecutive numbers

To find Sum of Consecutive numbers

1. Consider the following diagram:

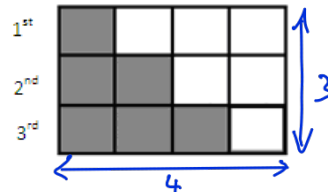


How many black boxes are there in

- a) the first row : (1)
the second row : (2)
- b) Total number of black boxes = (1) + (2)
- c) How many black boxes are there altogether?
(Note that exactly half of the squares are black)

Total number of black squares = $\frac{1}{2} \times (2) \times (3)$
- d) In conclusion: $1 + 2 = \frac{1}{2} \times (2) \times (3)$

2. Consider the following diagram:

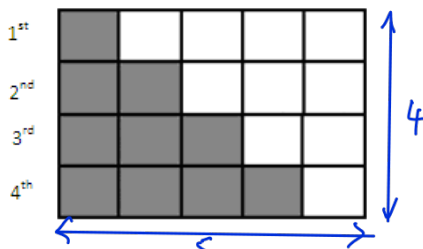


How many black boxes are there in

- a) the first row : (1)
the second row : (2)
the third row : (3)
- b) Total number of black boxes = (1) + (2) + (3)
- c) How many black boxes are there altogether?
(Note that exactly half of the squares are black)

Total number of black squares = $\frac{1}{2} \times (3) \times (4)$
- d) In conclusion: $1 + 2 + 3 = \frac{1}{2} \times (3) \times (4)$

3. Consider the following diagram:



How many black boxes are there in

- a) the first row : (1)
the second row : (2)
the third row : (3)
the fourth row : (4)
- b) Total number of black boxes
= (1) + (2) + (3) + (4)
- c) How many black boxes are there altogether?
(Note that exactly half of the squares are black)

Total number of black squares = $\frac{1}{2} \times (4) \times (5)$
- d) In conclusion:
 $1 + 2 + 3 + 4 = \frac{1}{2} \times (4) \times (5)$

4. Complete the following:

$1 + 2$	$= \frac{1}{2} \times (2) \times (3)$
$1 + 2 + 3$	$= \frac{1}{2} \times (3) \times (4)$
$1 + 2 + 3 + 4$	$= \frac{1}{2} \times (4) \times (5)$
$1 + 2 + 3 + 4 + 5$	$= \frac{1}{2} \times (5) \times (6)$
$1 + 2 + 3 + 4 + 5 + \dots + 100$	$= \frac{1}{2} \times (100) \times (101)$
$1 + 2 + 3 + 4 + 5 + \dots + \text{(last digit)}$	$= \frac{1}{2} \times (N) \times (N + 1)$

Important:

Sum of Consecutive number:
(starting with 1)

$1 + 2 + 3 + \dots + \text{(Last digit)}$

$= \frac{1}{2} \times N \times (N + 1)$

GUIDED EXAMPLE 1

Sum of Consecutive numbers

- a) (Consecutive numbers starting with 1)

$$1 + 2 + 3 + 4 + 5 + \dots + 400 = \frac{1}{2} \times 400 \times 401$$

$$= 80\,200$$

- b) (Even numbers)

$$2 + 4 + 6 + 8 + 10 + \dots + 400$$

$$= 2(1 + 2 + \dots + 199 + 200)$$

$$= 2 \times \frac{1}{2} \times 200 \times 201$$

$$= 40\,200$$

1	2	3	...	199	200	} 20100
1	2	3	...	199	200	
2	4	6		398	400	

P5 Heuristics Approach to Problem Solving

Pattern (I)

c) (Odd numbers)

$$1 + 3 + 5 + 7 + \dots + 99 = 2500$$

Sum	no. terms		Total
1	1	$\frac{1+1}{2}$	1 = 1x1
1+3	2	$\frac{1+3}{2}$	4 = 2x2
1+3+5	3	$\frac{1+5}{2}$	9 = 3x3
1+3+5+7	4	$\frac{1+7}{2}$	16 = 4x4
1+3+5+7+...+99	50	$\frac{1+99}{2}$	2500 = 50x50

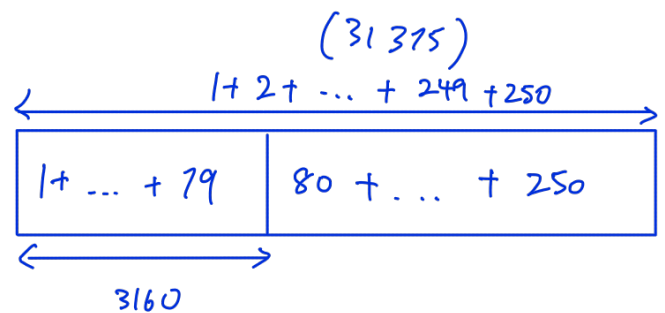
d) (Consecutive numbers not starting with 1)

$$80 + 81 + 82 + 83 + \dots + 250 = 28\,215$$

$$\begin{aligned} &1 + 2 + \dots + 249 + 250 \\ &= \frac{1}{2} \times 250 \times 251 \\ &= 31\,375 \end{aligned}$$

$$\begin{aligned} &1 + 2 + \dots + 78 + 79 \\ &= \frac{1}{2} \times 79 \times 80 \\ &= 3160 \end{aligned}$$

$$31\,375 - 3160 = 28\,215$$



e) (Consecutive numbers of multiples)

i. $10 + 20 + 30 + 40 + \dots + 1500$

$$= 10(1 + 2 + \dots + 149 + 150)$$

$$= 10 \times \frac{1}{2} \times 150 \times 151$$

$$= 113\,250$$

ii. $3 + 6 + 9 + 12 + \dots + 450$

$$= 3(1 + 2 + \dots + 149 + 150)$$

$$= 3 \times \frac{1}{2} \times 150 \times 151$$

$$= 33\,975$$

GUIDED EXAMPLE 2

* Observe changes

Sum of Consecutive numbers

Sequence Number	1	2	3	4
Number of dots	1	3	6	10

Find the number of dots in

a) 8th diagram

$$\begin{aligned}
 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 &= \frac{1}{2} \times 8 \times 9 \\
 &= 36
 \end{aligned}$$

b) 200th diagram

$$\begin{aligned}
 1 + 2 + \dots + 199 + 200 &= \frac{1}{2} \times 200 \times 201 \\
 &= 20100
 \end{aligned}$$

GUIDED EXAMPLE 3

A series of the sums of odd numbers is shown below.

1	=	1	=	1x1
1+3	=	4	=	2x2
1+3+5	=	9	=	3x3
1+3+5+7	=	16	=	4x4
1+3+5+7+9	=	25	=	5x5

Find the sum of the odd numbers from 1 to 51 inclusive.

$$1 + 3 + 5 + \dots + 47 + 49 + 51$$

$$\begin{aligned} \text{No. terms} &= \frac{51+1}{2} \\ &= 26 \end{aligned}$$

$$\begin{aligned} \text{Required sum} &= 26 \times 26 \\ &= 676 \quad (\text{Ans}) \end{aligned}$$

GUIDED EXAMPLE 4

A series of the sums of even numbers is shown below.

2	=	2	=	1x2
2+4	=	6	=	2x3
2+4+6	=	12	=	3x4
2+4+6+8	=	20	=	4x5
2+4+6+8+10	=	30	=	5x6

Find the sum of even numbers from 2 to 200 inclusive.

$$\begin{aligned}
 & 2 + 4 + \dots + 198 + 200 \\
 &= 2 (1 + 2 + \dots + 99 + 100) \\
 &= \cancel{2} \times \frac{1}{\cancel{2}} \times 100 \times 101 \\
 &= 10\ 100 \quad (\text{Ans})
 \end{aligned}$$

GUIDED EXAMPLE 5

Listing

The number of coins was observed to increase in a pattern as shown in the table below. Look for pattern and answer the following:

Week	1	2	3	4	5	6	7	8	9
Number of coins	5	10	20	40	80	160	320	640	1280

Handwritten annotations: Blue arrows with 'x2' below them connect each cell to the next in the 'Number of coins' row, indicating a doubling pattern.

a) How many coins will there be in week 6?

160

b) How many coins will there be in week 9?

1280

c) In which week will there be 320 coins?

Week 7

d) If the piggy bank is completely filled in week 15, during which week is the piggy bank $\frac{1}{2}$ filled with coins?

Fraction filled

Week 14 $\frac{1}{2}$
Week 15 1 whole

↙ x2

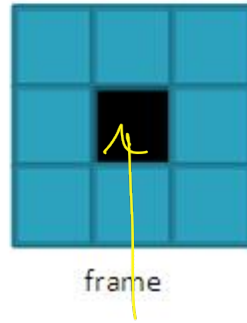
Ans: Week 14

GUIDED EXAMPLE 6

1. Table 1 below consists of number 1 to 56. Alice and Lynn are given a plastic frame that covers exactly 9 squares of Table 1 with the centre square darken.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56

Table 1



Average

Alice puts the frame on 9 squares as shown below:

3	4	5
11	12	13
19	20	21

- a) What is the average of the 8 numbers that can be seen on the frame?

$$(3 + 4 + 5 + 11 + 13 + 19 + 20 + 21) \div 8 = 12 \text{ (Ans)}$$

- b) Lynn puts the frame on some other 9 squares.
The sum of the 8 squares that can be seen in the frame is 272.
What is the largest number that can be seen in the frame?

$$\begin{aligned} \text{Average} &= \frac{272}{8} \\ &= 34 \end{aligned}$$

$$\begin{aligned} \therefore \text{Largest no.} &= 34 + 9 \\ &= 43 \text{ (Ans)} \end{aligned}$$

BUILD YOUR UNDERSTANDING

1. Find the sum of the following:

a) $1 + 2 + 3 + 4 + 5 + \dots + 150 = \frac{1}{2} \times 150 \times 151$
 $= 11\ 325$

b) $2 + 4 + 6 + 8 + \dots + 200 = 2(1 + 2 + \dots + 99 + 100)$
 $= 2 \times \frac{1}{2} \times 100 \times 101$
 $= 10\ 100$

c) $1 + 3 + 5 + 7 + 9 + \dots + 99$ #
 No. terms $= \frac{99+1}{2} = 50$ Required Sum $= 50 \times 50 = 2500$

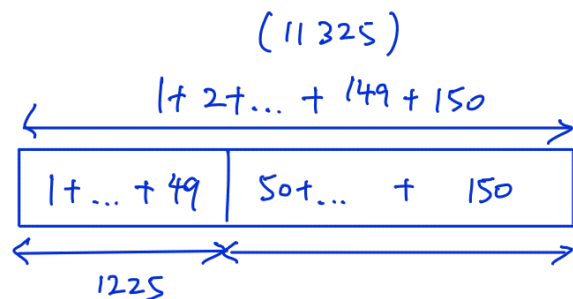
d) $4 + 8 + 12 + 16 + 20 + \dots + 600$
 $= 4(1 + 2 + \dots + 149 + 150)$
 $= 4 \times \frac{1}{2} \times 150 \times 151$
 $= 45\ 300$

e) $50 + 51 + 52 + 53 + \dots + 150 =$

$1 + 2 + \dots + 149 + 150 = 11\ 325$

$1 + 2 + \dots + 48 + 49 = \frac{1}{2} \times 49 \times 50$
 $= 1225$

$11\ 325 - 1225 = 10\ 100$



$$\begin{aligned}
 \text{f) } & (1+2+3+\dots+98+99)(99+98+\dots+2+1) \\
 & = 2(1+2+\dots+98+99) \\
 & = \cancel{2} \times \frac{1}{\cancel{2}} \times 99 \times 100 \\
 & = 9900
 \end{aligned}$$

$$\begin{aligned}
 \text{g) } & (1+2+3+\dots+200)(200+199+198+\dots+2+1) \\
 & = 2(1+2+\dots+199+200) \\
 & = \cancel{2} \times \frac{1}{\cancel{2}} \times 200 \times 201 \\
 & = 40200
 \end{aligned}$$


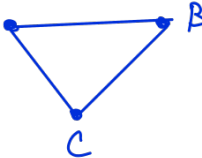
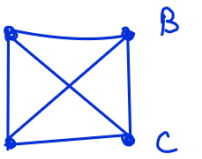
$$\begin{aligned}
 \text{h) } & \underbrace{(1+2+3+\dots+98+99)}_{\frac{1}{2} \times 99 \times 100} + \underbrace{(98+97+96+\dots+2+1)}_{\frac{1}{2} \times 98 \times 99} \\
 & = 4950 + 4851 \\
 & = 9801
 \end{aligned}$$

$$\begin{aligned}
 \text{i) } & (1+2+3+\dots+200)(199+198+\dots+2+1) \\
 & = \frac{1}{2} \times 200 \times 201 + \frac{1}{2} \times 199 \times 200 \\
 & = 40000
 \end{aligned}$$

P5 Heuristics Approach to Problem Solving

Pattern (I)

2. How many different handshakes can be made in a party of;

	No. children	No. HS	Re-write
a) 2 children? 	2	1 (a)	1
b) 3 children? 	3	+2 3 (b)	1 + 2
c) 4 children? 	4	+3 6 (c)	1 + 2 + 3
d) 100 children?	100	$\frac{1}{2} \times 99 \times 100$ $= 4950$ (d)	$1 + 2 + \dots + 98 + 99$
	50 (e)	1225	$1 + 2 + \dots + 48 + 49$

e) Given that there a total of 1225 handshakes made in the party, how many children are there in the party?
(Assume that each child shakes hands with every other child only once)

Trial & error

$\frac{1}{2} \times N \times (N+1) =$	Product (1225)	check
$\frac{1}{2} \times 20 \times 21 =$	210	X
$\frac{1}{2} \times 30 \times 31 =$	465	X
$\frac{1}{2} \times 40 \times 41 =$	820	X
$\frac{1}{2} \times 50 \times 51 =$	1275	X
$\frac{1}{2} \times 49 \times 50 =$	1225	✓

$$49 + 1 = 50 \text{ (Ans)}$$

5.

Sequence Number	1	2	3	4
Number of dots	1	3	6	10

Find the number of dots in

a) 8th diagram

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$$

b) 200th diagram

$$1 + 2 + \dots + 199 + 200 = 20100$$

P5 Heuristics Approach to Problem Solving

Pattern (I)

6. Study the pattern below. *even*
M4

Row							
1				2	3	4	5
2	9	8	7	6			
3				10	11	12	13
4	17	16	15	14			
5				18	19	20	21
6	25	24	23	22			
7				26	27	28	29
8	33	32	31	30			
9				34	35	36	37
10	(41)	(40)	(39)	(38)			
.....							
(14)	57	56	55	54			
.....						
200	(801)	(800)	(799)	(798)			
.....							
401				(1602)	(1603)	(1604)	(1605)

(a) Complete the pattern for Row 10 by filling up the four numbers in the above boxes.

57, 56, 55, 54

(b) If the four numbers in a row are ~~57, 58, 59~~ and 60, what is the Row number?

(c) What are the four numbers in Row 200?

(d) What are the four numbers in Row 401?

P5 Heuristics Approach to Problem Solving

Pattern (I)

8. Consider the following sums:

Line 1 4, 5 → 6, 7, 8
 Line 2 9, 10, 11 → 12, 13, 14, 15
 Line 3 16, 17, 18, 19 → 20, 21, 22, 23, 24

No. terms
 $5 = 5$
 $+2 \curvearrowright 7 = 5 + 1 \times 2$
 $+2 \curvearrowright 9 = 5 + 2 \times 2$
 $151 = 5 + \underline{73} \times 2$

a) Write down all the numbers in Line 4

Line 4 25, 26, 27, 28, 29 → 30, 31, 32, 33, 34, 35

b) If line 1, (that is 4, 5 → 6, 7, 8) is said to have 5 numbers, and line 2 has 7 numbers, how many numbers does Line 60 have?

$$5 + 59 \times 2 = 123$$

c) Which line has 151 numbers?

$$151 - 5 = 146$$

$$146 \div 2 = 73$$

$$73 + 1 = 74$$

