P5 HAPS – Lesson 4
Patterns (II)
Figures

enquiry@mathsheuristics.com
http://unittransfermethod.blogspot.com

LESSON 4: FIGURE PATTERNS

GUIDED EXAMPLE 1 _____ Sum of consecutive Numbers

Study the pattern below and complete the table:

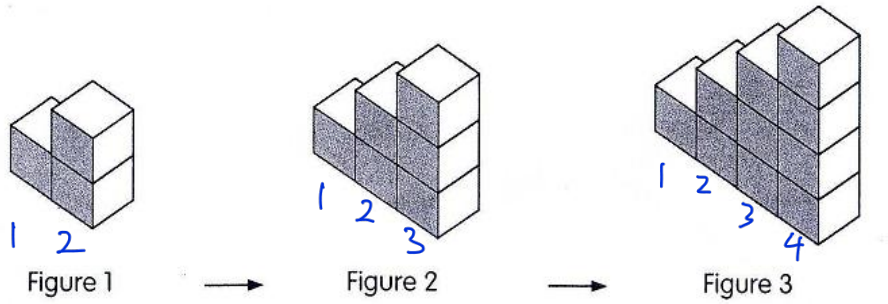


Figure 1 → Figure 2 → Figure 3

(a) Complete the following table:

Figure number	Number of cubes	
1	3	1+2
2	6	1+2+3
3	10	1+2+3+4
4	15	1+2+3+4+5
5	21	
100	5151	1+2+... +101

(b) Calculate the number of cubes in Figure 15.

$1+2+\dots+15+16 = \frac{1}{2}\times 16\times 17 = 136$

(c) Calculate the number of cubes in Figure 50.

$1+2+\dots+50+51 = \frac{1}{2}\times 51\times 52 = 1326$

(d) Which Figure number has 5151 cubes?

Trial & Error

$101-1=100$

$\frac{1}{2}\times \_\_ \times (\_\_ +1) = 5151$

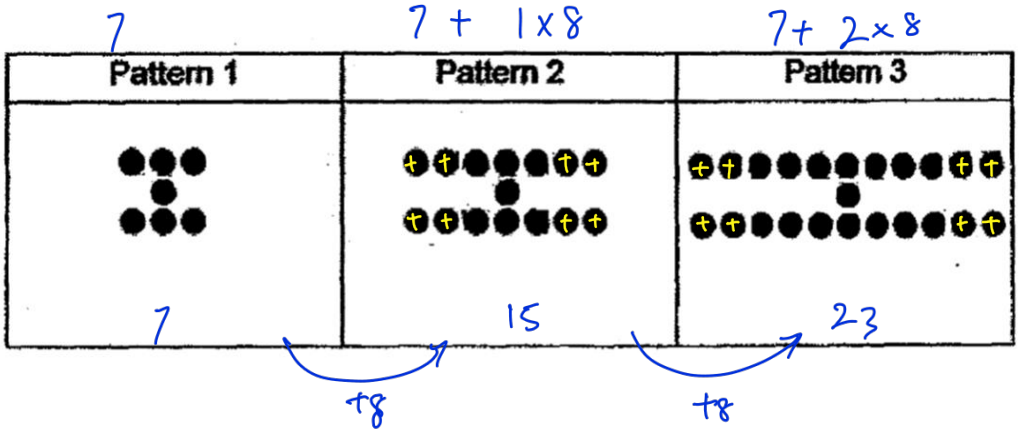
$\frac{1}{2}\times 90\times 91 = 4095$ ×

$\frac{1}{2}\times 100\times 101 = 5050$ ×

$\frac{1}{2}\times 101\times 102 = 5151$ ✓

GUIDED EXAMPLE 2 _____ Common Difference

Study the pattern carefully and answer the questions that follow.



a) How many dots are there in Pattern 5?

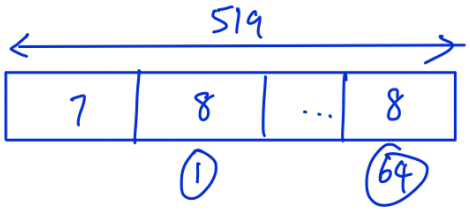
$23 + 8 + 8 = 39$

b) How many dots are there in pattern 73?

$7 + 72 \times 8 = 583$

519

c) What Pattern No. has ~~517~~ dots?



$7 + \underline{64} \times 8 = 519$

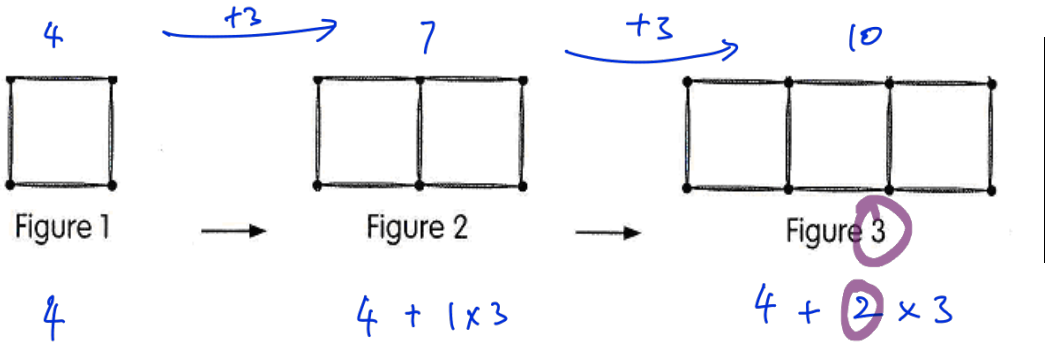
$519 - 7 = 512$

$512 \div 8 = 64$

$64 + 1 = 65$

GUIDED EXAMPLE 3 _____ Common Difference

Study the pattern below and answer the following questions:



4 4 + 1x3 4 + 2 x 3

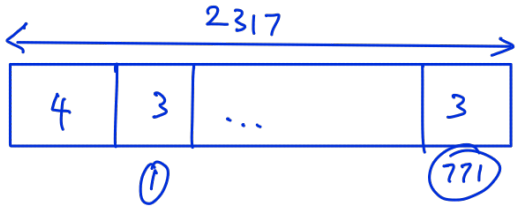
a) How many sticks will figure 5 have?

10 + 3 + 3 = 16

b) How many sticks will figure 91 have?

4 + 90 x 3 = 274

c) Which figure number has 2317 sticks?



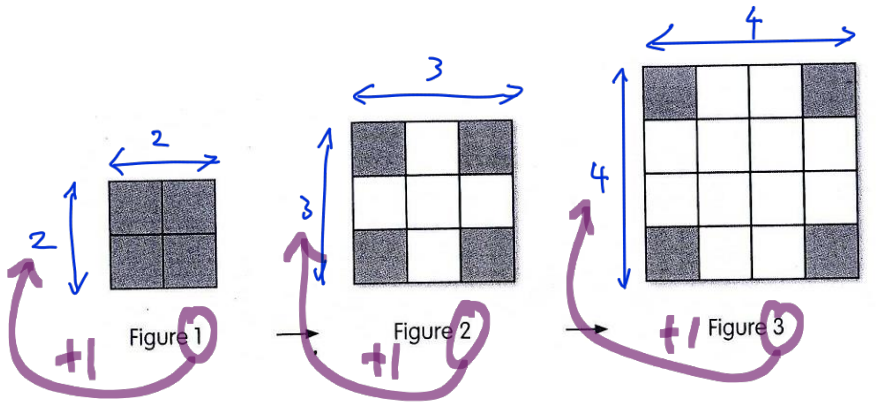
2317 - 4 = 2313

2313 ÷ 3 = 771

771 + 1 = 772

GUIDED EXAMPLE 4 _____ Square Numbers

Study the pattern below and fill up the table:



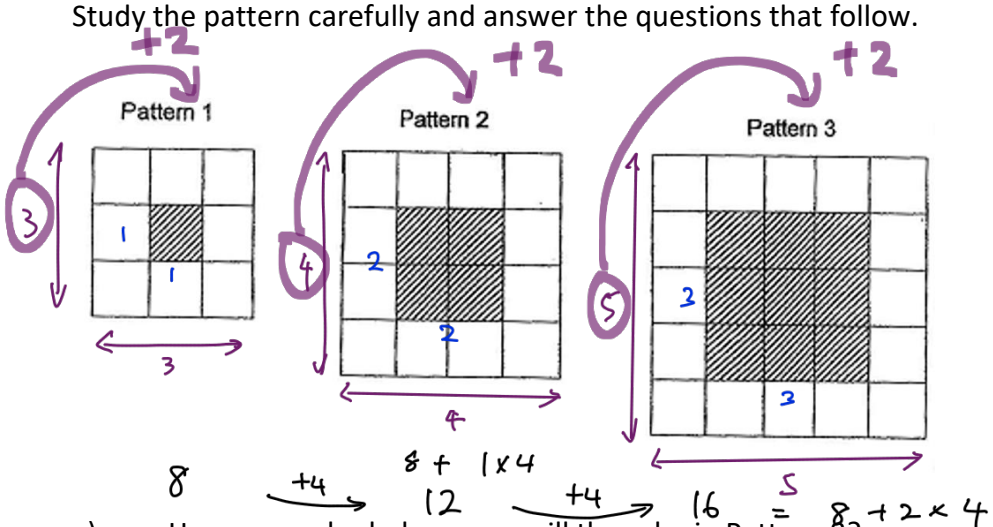
Complete the following table:

Total − Black

Figure	Number of black squares	Number of white squares	Total number of squares	
1	(4)	(0)	(4)	2×2
2	(4)	(5)	(9)	3×3
3	(4)	(12)	(16)	4×4
4	(4)	(21)	(25)	5×5
5	(4)	(32)	(36)	6×6
⋮	⋮	⋮	⋮	
10	(4)	(117)	(121)	11×11
⋮	⋮	⋮	⋮	
(11)	(4)	140	(144)	12×12
⋮	⋮	⋮	⋮	
(24)	(4)	(621)	625	25×25

GUIDED EXAMPLE 5 Square Numbers

Study the pattern carefully and answer the questions that follow.



a) How many shaded squares will there be in Pattern 8?

$8 \times 8 = 64$

b) How many unshaded squares are there in Pattern 14?

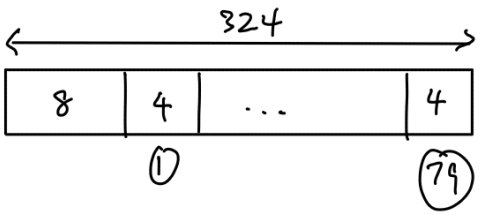
$16 \times 16 - 14 \times 14 = 60$

c) What pattern number has 324 unshaded squares?

$324 - 8 = 316$

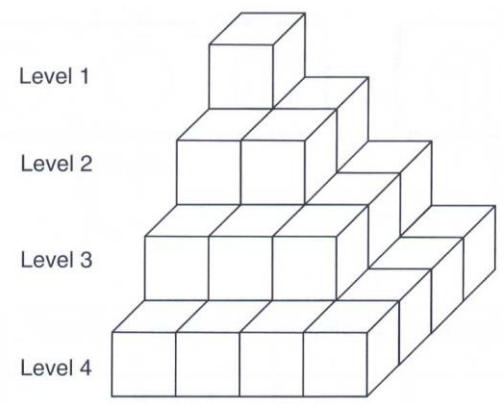
$316 \div 4 = 79$

$79 + 1 = 80$



GUIDED EXAMPLE 6 Systematic Listing

Study the pattern of the structure below.



(a) Fill in the missing number of cubes.

Level	Number of cubes	Total
1	(1)	(1)
2	(4)	(5)
3	(9)	(14)
4	(16)	(30)
5	(25)	(55)
:	:	:
10	(100)	6) (385)
11	11x11 = 121	385 + 121 = 506

6 x 6 = 36
7 x 7 = 49
8 x 8 = 64
9 x 9 = 81
10 x 10 = 100

(b) If more cubes are to be added to build the structure, how many cubes are needed to use for Level 10?

55 + 36 + 49 + 64 + 81 + 100 = 385

(c) If a total of 506 cubes are used to build the whole structure, how many levels are there in the structure?

Listing

Ans : 11

GUIDED EXAMPLE 7 Cyclical Patterns

See the pattern below.

R1 R2 No R

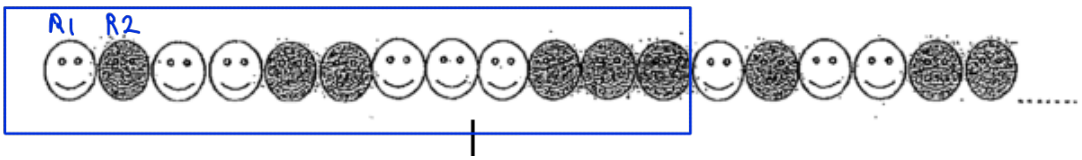
What is the 79th shape in the pattern?

(MGS P5 CA1 2014)

$79 \div 3 = 26 \text{ R } 1$

Ans : △

A group of pupils were given white and grey 'Smiley face' badges.
They are arranged in the patten shown below.



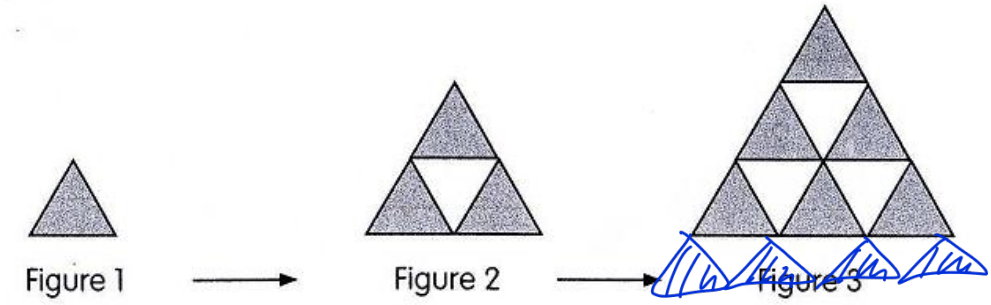
If there are 74 badges, what is the colour of the last badge?
(Rosyth P5 SA2 2012)

$74 \div 12 = 6 \text{ R } 2$

Ans : Grey

BUILD YOUR UNDERSTANDING

1. Study the pattern below.
Note: The white space inside the figure is empty



(a) Complete the following table:

Figure	Number of (shaded) triangles
1	(1)
2	(3)
3	(6)
4	(10)
5	(15)

+2, +3, +4, +5

1
1 + 2
1 + 2 + 3
1 + 2 + 3 + 4
1 + 2 + 3 + 4 + 5

(a) Calculate the number of triangles in Figure 25.

$1 + 2 + \ldots + 24 + 25 = \frac{1}{2} \times 25 \times 26 = 325$

(b) Calculate the number of triangles in Figure 40.

$1 + 2 + \ldots + 39 + 40 = \frac{1}{2} \times 40 \times 41 = 820$

(c) Which Figure number has 595 triangles?

$\frac{1}{2} \times 30 \times 31 = 465$ ✗

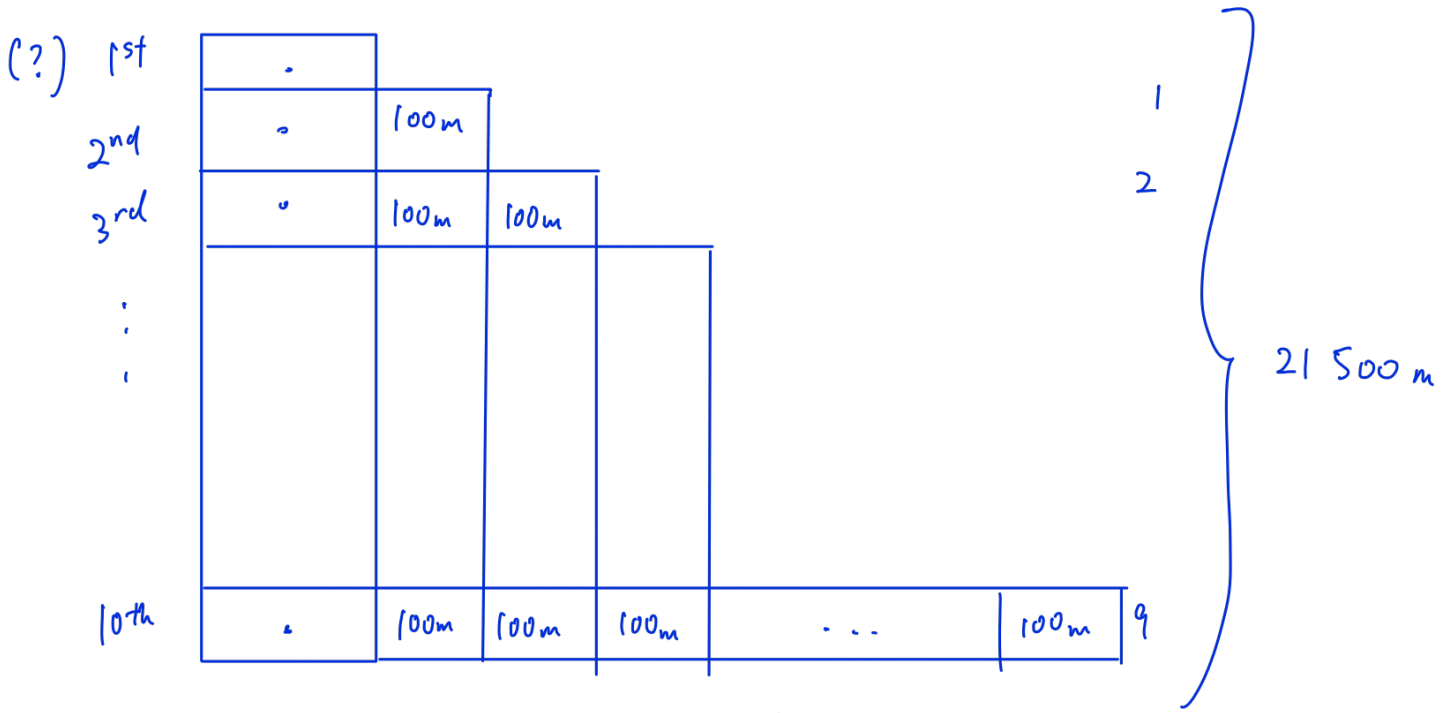
$\frac{1}{2} \times 35 \times 36 = 630$ ✗

$\frac{1}{2} \times 34 \times 35 = 595$ ✓

$= 1 + 2 + \ldots + 33 + 34$

Ans : 34

2. Su Lin is training for a running event.
For each day after the first day, she ran 100 m more than the previous day.
At the end of the 10 days, she ran a total of 21500 m.
How far did she run on the first day?



$1 + 2 + \ldots + 8 + 9 = \frac{1}{2} \times 9 \times 10$

$= 45$

$21\,500 - 45 \times 100 = 17\,000$

$17\,000 \div 10 = 1\,700$

Ans : 1700 m

3. Study the pattern below.

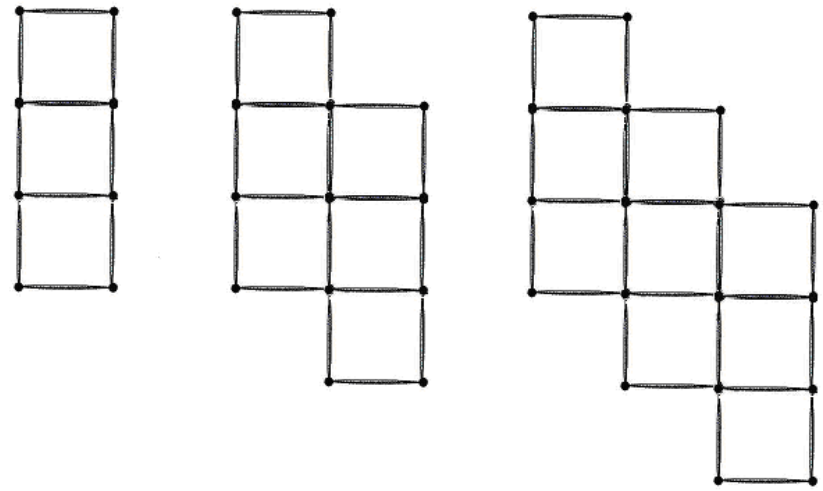


Figure 1 → Figure 2 → Figure 3

Each □ represents a square. Complete the following table.

Figure	Number of Squares	Number of sticks	
1	(3)	(10)	10
2	(6)	(18)	10 + 1×8
3	(9)	(26)	10 + 2×8
4	(12)	(34)	10 + 3×8
5	(15)	(42)	10 + 4×8
:	:	:	
50	(150)	(402)	10 + 49×8
:	:	:	
(100)	300	(802)	10 + 99×8
:	:	:	
(111)	(333)	890	10 + 110×8

×3 (Number of Squares: 3 → 6 → 9 → 12 → 15)

+8 (Number of sticks: 10 → 18 → 26 → 34 → 42)

÷3 (Number of Squares: 300 → 100)

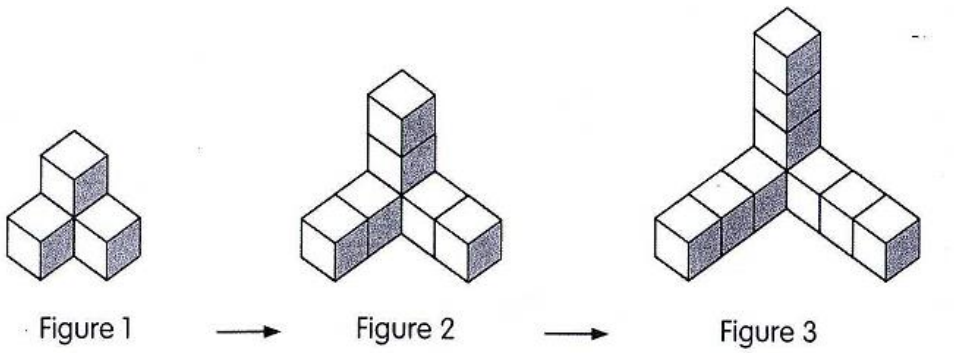
×3 (Number of Squares: 111 → 333)

890 − 10 = 880

880 ÷ 8 = 110

110 + 1 = 111

4. Study the pattern below.



Complete the following table.

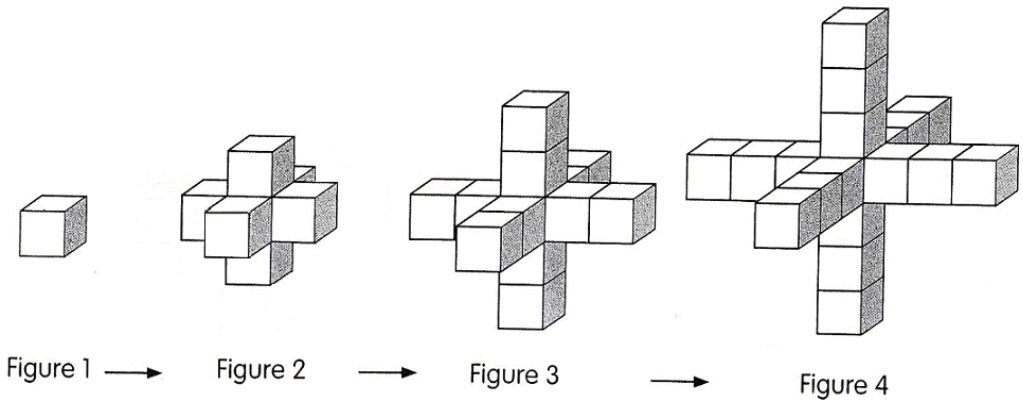
Figure	Number of cubes
1	(4) 4
2	+3 (7) 4+1x3
3	+3 (10) 4+2x3
4	+3 (13) 4+3x3
5	+3 (16) 4+4x3
:	:
30	(91) 4+29x3
:	:
60	(181) 4+59x3
:	:
(90)	271 4+89x3

271 − 4 = 267

267 ÷ 3 = 89

89 + 1 = 90

5. Study the pattern below.



Complete the following table.

Figure	Number of cubes
1	(1) 1
2	(7) 1+1x6
3	(13) 1+2x6
4	(19) 1+3x6
:	:
20	(115) 1+19x6
:	:
50	(295) 1+ 49x6
:	:
(80)	475 1+79x6

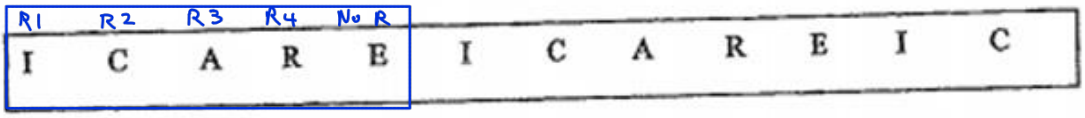
475 - 1 = 474

474 ÷ 6 = 79

79 + 1 = 80

6. Judy uses the five letters of I,C,A,R and E to form a pattern.

space



The first 12 letters are shown below.
Which letter is in the 37th position?

(Nan Hua P5 CA1)

37 ÷ 5 = 7 R 2

Ans: C

7. Some shapes are arranged in the following pattern:

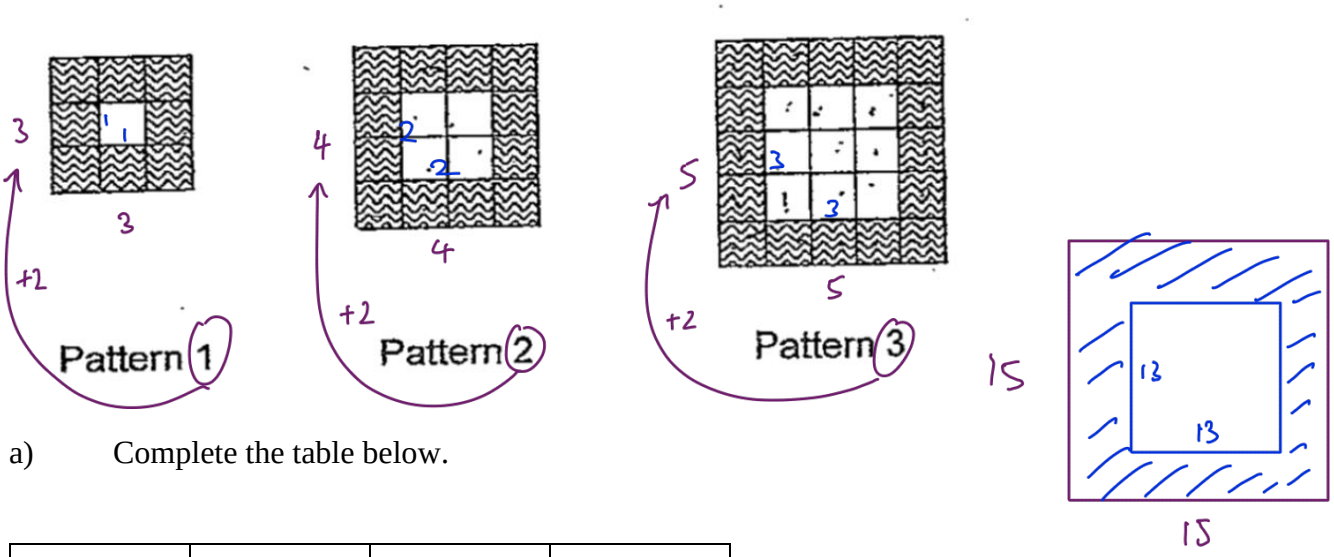
Which shape is the 59th position?

(MGS P5 SA1 2014)

59 ÷ 4 = 14 R 3

Ans: ○

8. A wall design is created using white and shaded tiles.



a) Complete the table below.

Pattern No.	Shaded	Unshaded	Total	
1	8	1	9	8
2	12	4	16	8 + 1×4
3	16	9	25	8 + 2×4
4	20	16	36	8 + 3×4
5	24	25	49	
10		100		

b) How many shaded tiles will there be in Pattern 13?

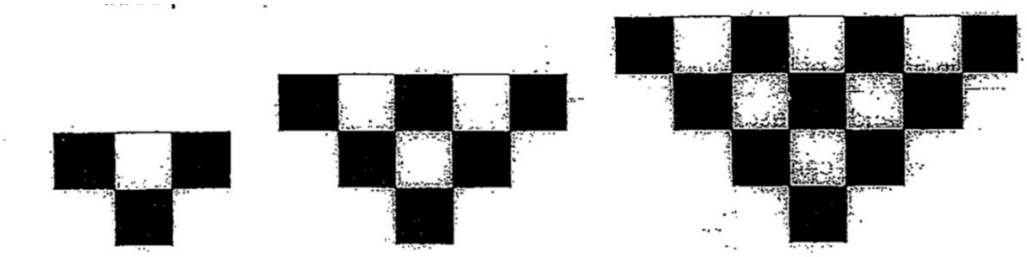
$15 \times 15 - 13 \times 13 = 56$

c) A pattern has 100 ~~white~~ Unshaded tiles. How many shaded ~~squares~~ tiles are there?

$\sqrt{100} = 10$

$12 \times 12 - 100 = 44$

9. Black and white squares are used to form a sequence of patterns.
The first three patterns are shown below.



Pattern 1 Pattern 2 Pattern 3

a) Complete the information in the table provided.

Pattern No.	Shaded	Unshaded	Total
1	3	1	$4 = 2\times2$
2	6 (+3)	3 (+2)	$9 = 3\times3$
3	10 (+4)	6 (+3)	$16 = 4\times4$
4	15 (+5)	10 (+4)	$25 = 5\times5$
5	21 (+6)	15 (+5)	$36 = 6\times6$
26			$729 = 27\times27$

Unshaded
1
1 + 2
1 + 2 + 3
1 + 2 + 3 + 4

b) Find the number of ~~white~~ Unshaded squares in Pattern 27.

$1 + 2 + \ldots + 26 + 27 = \frac{1}{2} \times 27 \times 28$

$= 378$

c) Which pattern number will there be 729 squares?

$\sqrt{729} = 27$

$27 - 1 = 26$