LESSON 8: Work Backwards

Work Backwards

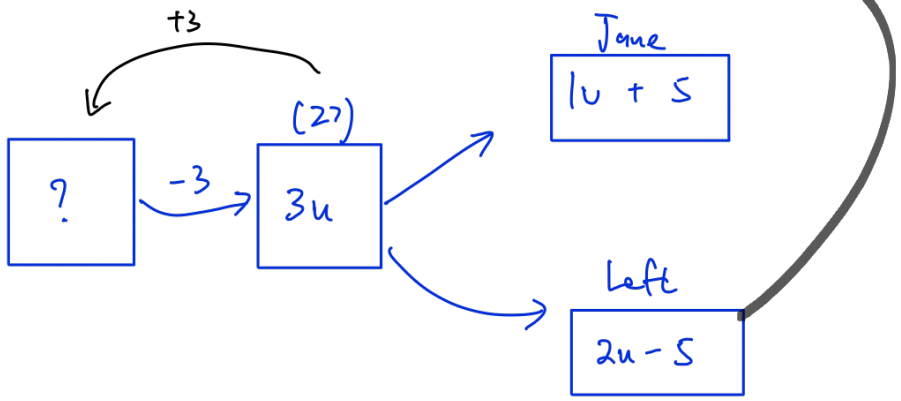
"Work Backwards" strategy is useful when the final result has already been given.

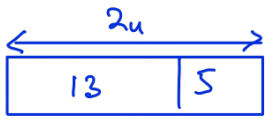
- For problems involving single event, the use of flow chart is recommended
- For problems involving multiple events, the use of tabulation is recommended

Type 1: Problems involving single event

GUIDED EXAMPLE 1

Linda bought some chocolate bars.
She ate 3 chocolate bars and divided the rest into 3 equal shares.
She gave 1 share and 5 more chocolate bars to Jane.
She was left with 13 chocolate bars.
How many chocolate bars did she have at first?





$2u - 5 = 13$

$2u = 13 + 5$

$= 18$

$3u = 3 \times \frac{18}{2}$

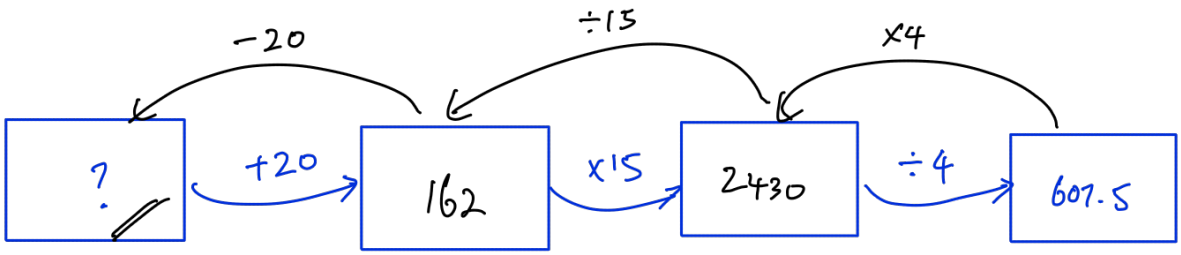
$= 27$

$27 + 3 = 30$

Ans : 30

GUIDED EXAMPLE 2

Kenny enters a number into his calculator.
He adds 20 to his number.
Then he multiplies the sum by 15.
Finally, he divides the product by 4.
If the final answer is 607.5,
what is the number Kenny ~~enters~~ entered into his calculator?



607.5 × 4 = 2430

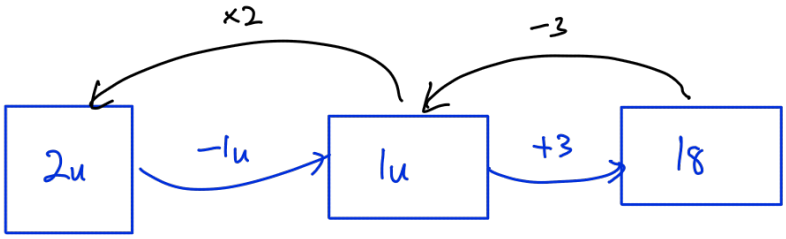
2430 ÷ 15 = 162

162 − 20 = 142

Ans : 142

GUIDED EXAMPLE 3

Alice had a certain number of marbles in the container.
She took $\frac{1}{2}$ of the marbles out of the box.
She then put 3 marbles back into the box and repeated this process five times.
If there were 18 marbles left in the in the end,
how many marbles were in the container at first?



5) (18 − 3) × 2 = 30

4) (30 − 3) × 2 = 54

3) (54 − 3) × 2 = 102

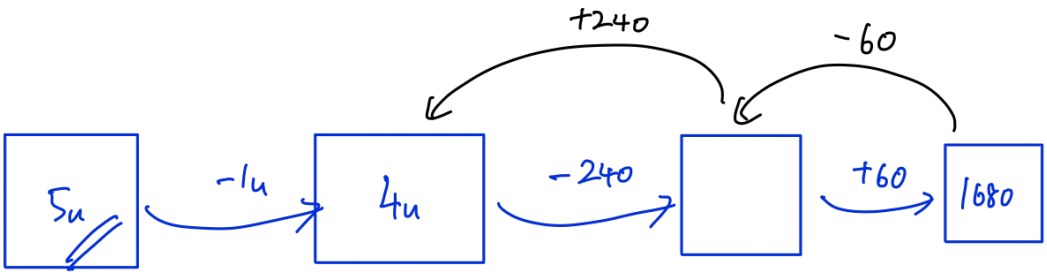
2) (102 − 3) × 2 = 198

1) (198 − 3) × 2 = 390

Ans : 390

GUIDED EXAMPLE 4

Sam bought some apples from the supplier.
He threw away ~~20~~ $\frac{1}{5}$ of them, (1u, 5u)
sold 240 apples and then bought another 60 apples.
If he had 1680 apples now, how many apples did he buy at first?



$4u = 1680 - 60 + 240$

$= 1860$

$5u = 5 \times \frac{1860}{4}$

$= 2325$

Ans : 2325

Type 2: Problems involving multiple events

GUIDED EXAMPLE 5 Double Events

There were a total of 360 marbles in container A and container B.
Ali transferred 20% of the marbles from container A to container B.
After, he transferred 40% of the marbles from container B back to container A.
In the end, the two containers have equal number of marbles.
How many marbles are there in each container at first?

	A	B	Total
B	5u ×15 → 75	285	360
C	−1u ×15 → −15	+1u ×15 → +15	
A	4u ×15 → 60	5p ×60 → 300	360
C	+2p ×60 → +120	−2p ×60 → −120	
A	180	3p ×60 → 180	360

* internal transfer → total unchanged
** End result known → work backwards.

360 ÷ 2 = 180

Ans: A : 75
B : 285

GUIDED EXAMPLE 6 Triple Events

B Sally had 234 beads in 3 containers, X, Y and Z.
C If she moved 25 beads from container X to container Y,
C 17 beads from container Y to container Z and
C 19 beads from container Z to container X,
A there would be an equal number of beads in each container.
How many beads were there in each container at first?

	X	Y	Z	Total
B	84	70	80	234
C	−25	+25		
C		−17	+17	
C	+19		−19	
A	78	78	78	234

$234 \div 3 = 78$

Ans: X : 84
Y : 70
Z : 80

GUIDED EXAMPLE 7

Triple Events

B There were a total of 340 marbles in Box A, Box B and Box C.

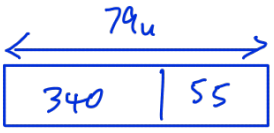
C Andy gave away half of the number of marbles in Box A,
tripled the number of marbles in Box B and put in additional 55 marbles in Box C.

A As a result, ratio of the number of marbles in Box A to Box B to Box C became 27:27:16.
Find the total number of marbles in the three boxes in the end.

	A	B	C	Total
B	2 ×27 → 54u	9u	16u − 55	340
C	−1 ×27 → −27u	×3	+55	
A	1 ×27 → 27u	27u	16u	70u

$54u + 9u + 16u - 55 = 340$

$79u - 55 = 340$



$79u = 340 + 55$

$= 395$

$1u = 395 \div 79$

$= 5$

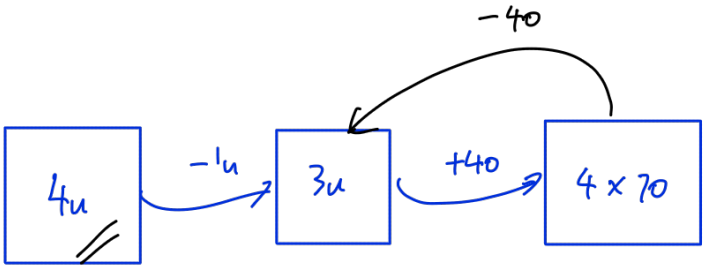
$70u = 70 \times 5$

$= 350$

Ans : 350

BUILD YOUR UNDERSTANDING!

1. Jenny had a certain number of books.
She donated $\frac{1}{4}$ to the orphanage and bought 40 new books.
She then divided all her books into 4 equal stacks and placed them on her shelf.
If she had 70 books in each stack, how many books did she have at first?



$3u = 4 \times 70 - 40$

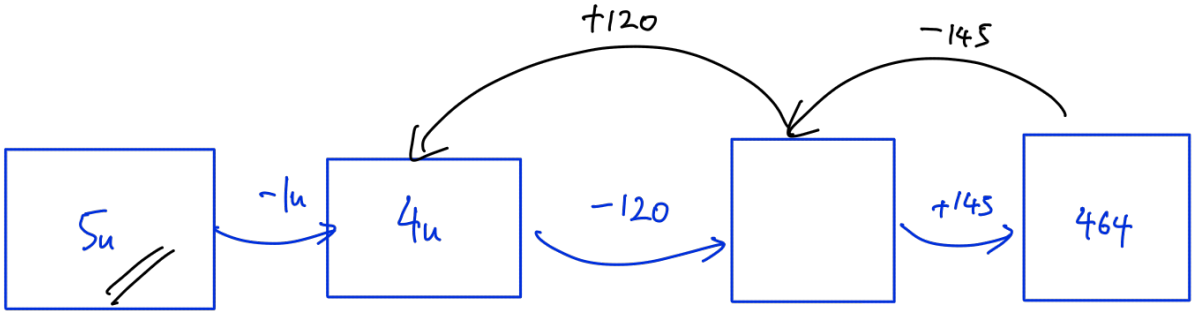
$= 240$

$4u = 4 \times \frac{240}{3}$

$= 320$

Ans : 320

2. Cindy received some money from her father.
She spent 20% of her savings on a purse and $120 on a blouse.
Her mother gave her $145. If she had $464 left,
how much money did she receive at first?



$4u = 464 - 145 + 120$

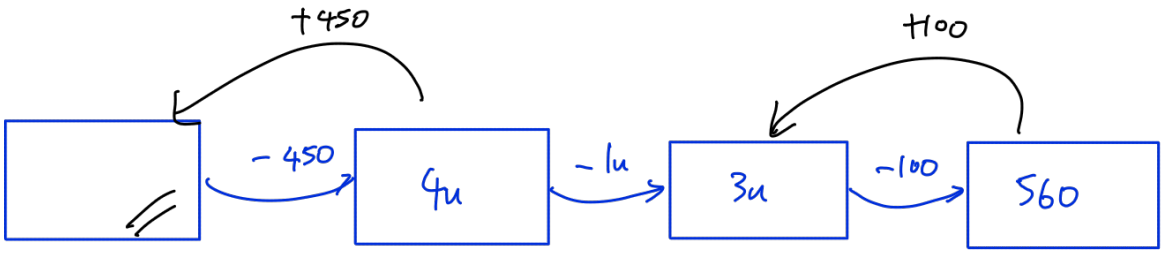
$= 439$

$5u = 5 \times \frac{439}{4}$

$= 548.75$

Ans : $548.75

3. Ali had won some money in the lottery draw.
He spent $450 and gave 25% of his remaining money and $100 to his sister.
In the end, he was left with $560.
How much money did he win?



$3u = 560 + 100$

$= 660$

$4u = 4 \times \frac{660}{3}$

$= 880$

$880 + 450 = 1330$

Ans : $1330

4. Steve and Tim had a total of \$84.
Steve gave $\frac{1}{3}$ of his money to Tim.
Tim then gave $\frac{3}{5}$ of his money Steve.
Steve gave 25% of his money to Tim.
In the end, each of them had the same amount of money.
How much money had each of them at first?

	Steve	Tim	Total
B	3u ×7 → 21	63	84
C	−1u ×7 → −7	+1u ×7 → +7	
A	2u ×7 → 14	5p ×14 → 70	84
C	+3p → +42	−3p ×14 → −42	
A	4s ×14 → 56	2p ×14 → 28	84
C	−1s ×14 → −14	+1s ×14 → +14	
A	3s ×14 → 42	42	84

84 ÷ 2 = 42

Ans : Steve : \$21
Tim : \$63

5. Linda is arranging her dolls into two boxes, X and Y.
First, she transfers $\frac{1}{2}$ of her dolls in Box Y to Box X.
After, she transfers $\frac{1}{5}$ of her dolls in Box X to Box Y.
Subsequently, she transfers $\frac{1}{6}$ of her dolls in Box Y to Box X.
In the end, there are $\frac{3}{7}$ as many dolls in Box Y as that in Box X.

a) Find the ratio of the number of dolls in Box X to that in Box Y initially.

b) If there are 100 more dolls in Box X than Box Y in the end, how many dolls are there in Box X initially?

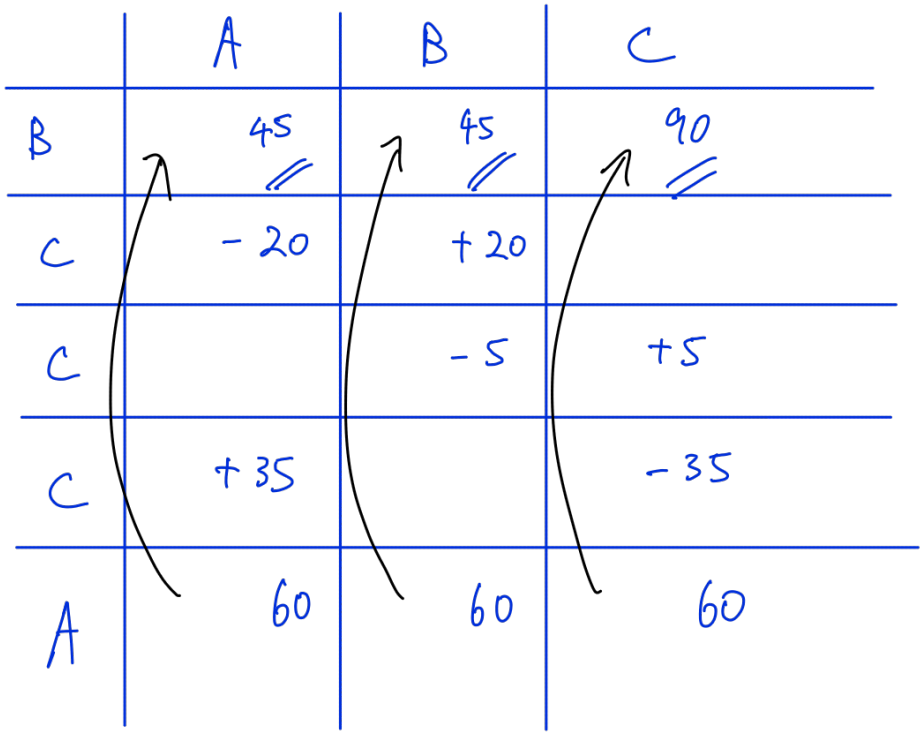
	X	Y	Total
B	(b) 60	2u ×20 40	100
C	+1u ×20 +20	-1u ×20 -20	
A	5p ×16 80	1u ×20 20	100
C	-1p ×16 -16	+1p ×16 +16	
A	4p ×16 64	6s ×6 36	100
C	+1s ×6 +6	-1s ×6 -6	
A	70	5s ×6 30	100

a) 60 : 40
= 3 : 2

b) 70 boxes − 30 boxes = 100
40 boxes = 100
60 boxes = $60 \times \frac{100}{40}$
= 150

Ans : a) 3 : 2
b) 150

6. There are some water in pail A, B and C.
When 20 litres of water was poured from pail A to pail B,
5 litres from pail B to pail C and 35 litres from pail C to pail A,
the three pails contained 60 litres of water each.
How much water was in each pail at first?



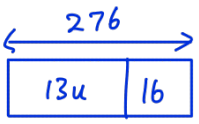
Ans: A : 45 ℓ
B : 45 ℓ
C : 90 ℓ

7. There was a total of 276 beans in Bottle A, Bottle B and Bottle C.
Philip removed $\frac{1}{3}$ of the beans from bottle A,
16 beans from bottle B and added more beans to bottle C until the number of beans in it was doubled.
As a result, the ratio of the number of beans in bottle A to bottle B to bottle C was then 4:5:4.
Find the number of beans in bottle B now.

	A	B	C	Total
B	3 ×2u → 6u	5u + 16	2u	276
C	−1 ×2u → −2u	−16	×2	
A	2 ×2u → 4u	5u	4u	

$6u + 5u + 16 + 2u = 276$

$13u + 16 = 276$



$13u = 276 - 16$

$= 260$

$1u = 260 \div 13$

$= 20$

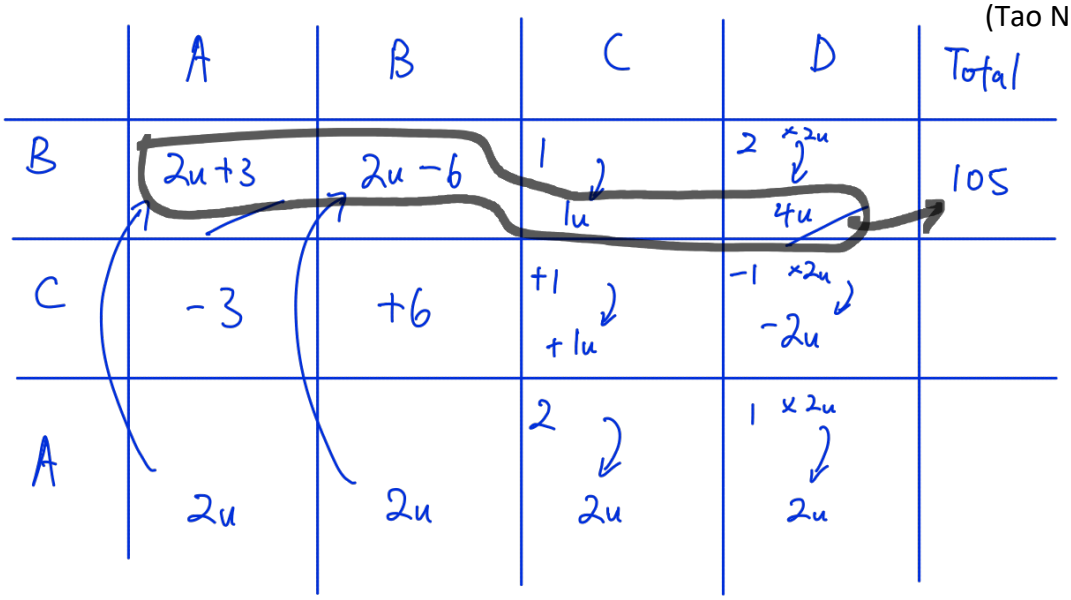
$5u = 5 \times 20$

$= 100$

Ans : 100

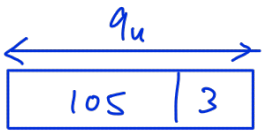
8. B Alan, Benny, Clement and Danny had 105 marbles.
C Alan lost 3 marbles, and Danny lost half of what he had.
Benny's sister gave Benny another 6 marbles.
Clement's aunt rewarded Clement by doubling what he had originally.
A In the end, the 4 boys had an equal number of marbles.
How many more marbles than Danny did Alan have at first?

(Tao Nan Pri/SA1/P5)



$2u+3+2u-6+1u+4u=105$

$9u-3=105$



$9u=105+3$

$=108$

$1u=108\div 9$

$=12$

$2u+3=2\times 12+3$

$=27$

$4u=4\times 12$

$=48$

$48-27=21$

Ans : 21