LESSON 9 Rate (I)

Note:

1. For problems relating to unknown amount of work, we often refer to 'whole' problem and use the number '1'.

2. There are three problems related to rate problems:

   ▪ The first type is 'Working together' problems.

   ▪ The second type is 'Working alone' problems.

   ▪ The third type is 'Leaking tap' problems.

GUIDED EXAMPLE 1 * make time the same Working Together

Leo can paint a room in 6 hours while
Benny can paint the same room in 3 hours.
How long will it take for both of them to paint
the same room working together?

	Time (h)	Room
L	6	1
B	3 ×2 → 6	1 ×2 → 2
L+B	6 ÷3 → 2 //	1 + 2 = 3 ÷3 → 1

Ans : 2h

GUIDED EXAMPLE 2

Working Together

3, 6, 9, 12
4, 8, 12

Ahmad can paint an apartment in 1 day.
Billy can paint 2 similar apartments in 3 days.
Charlie can paint 3 apartments in 4 days.
How long would it take the three of them working together to paint 58 apartments?

	Time (Days)	Apartment
A	1 ×12 → 12	1 ×12 → 12
B	3 ×4 → 12	2 ×4 → 8
C	4 ×3 → 12	3 ×3 → 9
A+B+C	12 ×2 → 24 //	12 + 8 + 9 = 29 ×2 → 58

Ans : 24 days

GUIDED EXAMPLE 3

Working Alone

Linden and Ben took 3 days to complete a project.
Linden could complete the project alone in 7 days.
How long would it take for Ben to complete the project alone?

	Time (Days)	Project
L+B	3 ×7 → 21	1 ×7 → 7
L	7 ×3 → 21	1 ×3 → 3
B	21 ÷4 → $5\frac{1}{4}$	7 − 3 = 4 ÷4 → 1

Ans: $5\frac{1}{4}$ days

GUIDED EXAMPLE 4 * vertically align *

When Ben partners Harry, they take 24 days to complete a project.
When Ken partners Ben, they take 8 days to complete the same project.
When Harry partners Mandy, they take 12 days to complete the same project.
How many days do Ken and Mandy take to complete the project if they worked together?

	Time (Days)	Project
B + H	24	1
B + K	8 ×3 → 24	1 ×3 → 3
M + H	12 ×2 → 24	1 ×2 → 2
M + B + H + K	24	3 + 2 = 5
M + K	24 ÷4 → 6	5 − 1 = 4 ÷4 → 1

Ans : 6 days

GUIDED EXAMPLE 5

Tom takes 20 days to complete a project.
Danny takes 28 days to complete the similar project.
If Tom starts the project alone first and leaves the remainder to Danny,
they will take 24 days in all to complete the project.
How many days does Tom spend on the project?

1 whole

Total No. days

Tom: 20 days → 1 project
1 day → $\frac{1}{20}$ project

Danny: 28 days → 1 project
1 day → $\frac{1}{28}$ project

Suppose that Danny works for all 24 days.

$24 \times \frac{1}{28} = \frac{24}{28}$

D: $1 - \frac{24}{28} = \frac{1}{7}$

d: $\frac{1}{20} - \frac{1}{28} = \frac{1}{70}$

D ÷ d: $\frac{1}{7} \div \frac{1}{70} = 10$

Replace 10 days from Danny with 10 days from Tom.

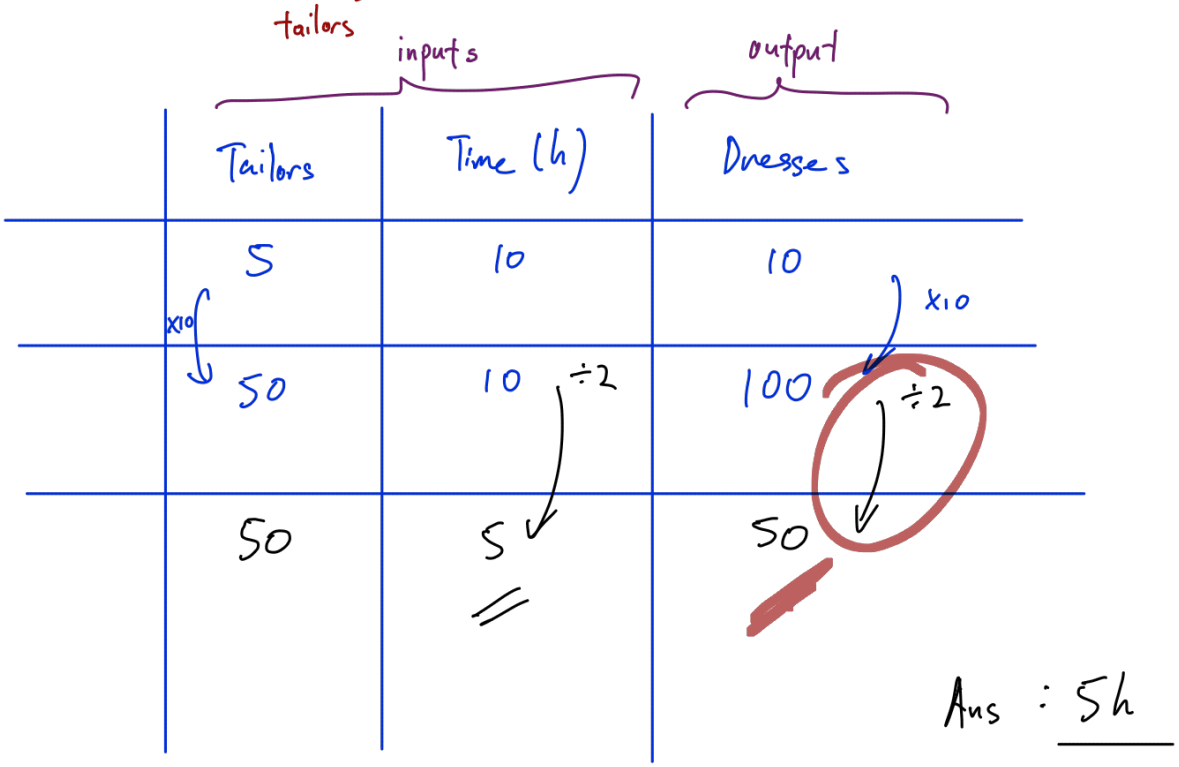
∴ No. days Tom spent = 10

Ans: 10 days

GUIDED EXAMPLE 6

Three Ratio

In a sewing factory, 5 ~~dressers~~ tailors can sew 10 dresses in 10 hours on average.
How long will 50 ~~dressers~~ tailors in the factory take to sew 50 dresses on average?



BUILD YOUR UNDERSTANDING

1. Machine A alone takes 12 hours to complete a printing job.
Machine A and B working together take 8 hours to complete the same job.
How many hours would it take Machine B to complete the job alone?

	Time (h)	Job
A	12 ×2 → 24	1 ×2 → 2
A + B	8 ×3 → 24	1 ×3 → 3
B	24	3 − 2 = 1

Ans: 24h

2. There are three old photocopier machines in the office.
In 4 minutes, the first machine can print 120 pages
which the second machine can print 160 pages.
The third photocopier can print 60 pages in 2 minutes.

(a) If all machines are used, how long will it take to print 4800 pages?

(b) If the first machine and the third machine break down after 32 minutes of printing, how long will it take to print the same number of pages?

	Time (min)	Pages
M1	4 ÷4 → 1	120 ÷4 → 30
M2	4 ÷4 → 1	160 ÷4 → 40
M3	2 ÷2 → 1	60 ÷2 → 30
M1+M2+M3	1 ×48 → a) 48 → 32	30+40+30 = 100 ×48 → 4800 → 3200

b) 4800 − 3200 = 1600
1600 ÷ 40 = 40
40 + 32 = 72

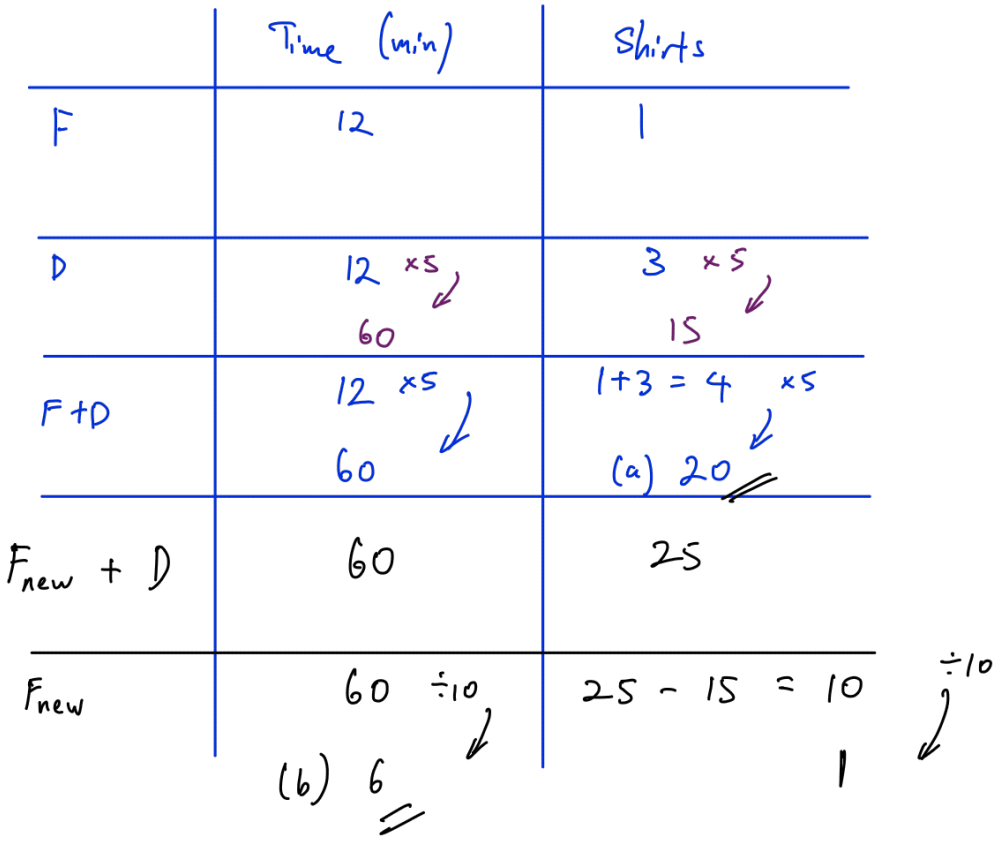
Ans: a) 48 min
b) 72 min

3. Fatimah and Devi sew shirts in a factory.
The average time Fatimah takes to sew a shirt is 12 minutes.
Devi can sew 3 shirts in the same amount of time.

a) Working together, how many shirts can Fatimah and Devi sew in 1 hour 60 min

b) If Fatimah sews faster and together they can sew 25 shirts in 1 hour,
find the average time taken by Fatimah to sew a shirt.

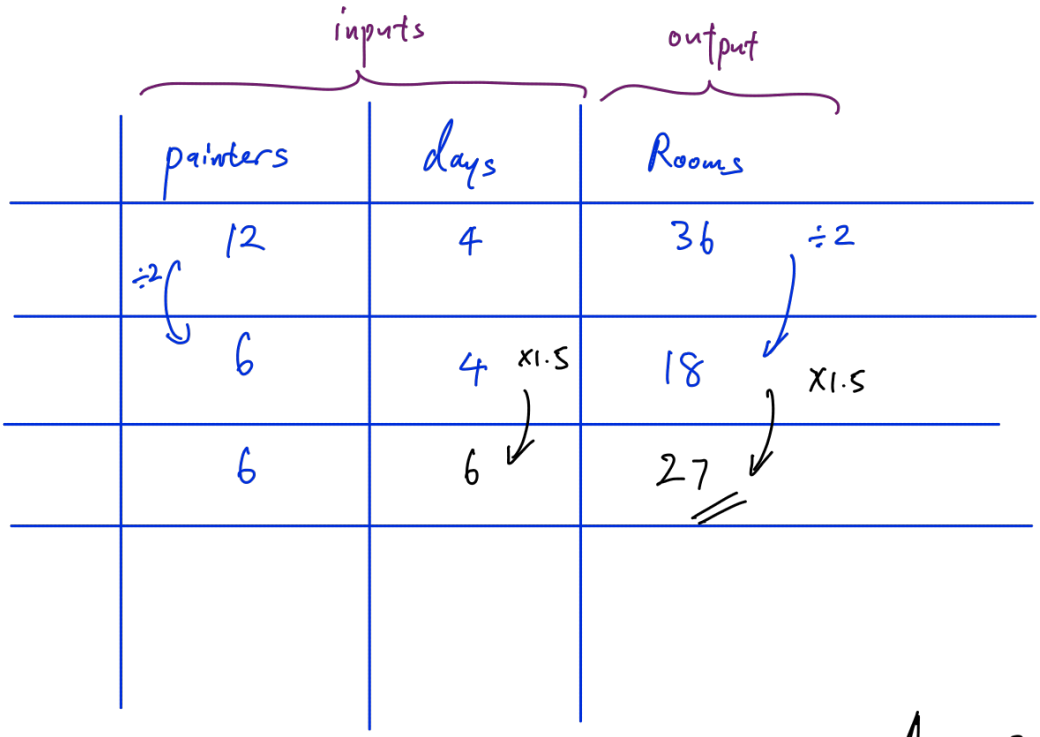
(Pei Chun Public School/SA2/Paper 2/Q45)



Ans: a) 20

b) 6 min

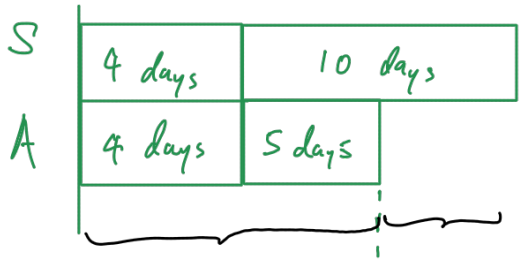
4. If 12 painters took 4 days to paint 36 rooms,
how many rooms will 6 painters complete in 6 days?



Ans: 27

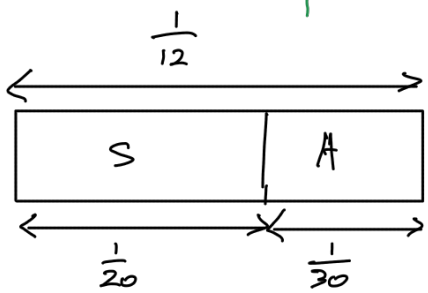
5. Sly and Ali will take 12 days to complete a project working together.
If the two boys first work together for 4 days,
followed by Sly working alone for 10 days,
Ali will take 5 days to complete painting the remaining project.
How long will each boy take to complete the project working alone?

	Days	Project
S + A	12 ÷12 → 1 ×9 → 9	1 ÷12 → $\frac{1}{12}$ ×9 → $\frac{9}{12} = \frac{3}{4}$
S	5 ×4 → 20 ÷20 → 1	$\frac{1}{4}$ ×4 → 1 ÷20 → $\frac{1}{20}$
A	1 ×30 → 30	$\frac{1}{12} - \frac{1}{20} = \frac{1}{30}$ ×30 → 1



S+A worked together for 9 days
$\frac{3}{4}$ of project

S worked alone for 5 days
$\frac{1}{4}$ of Project

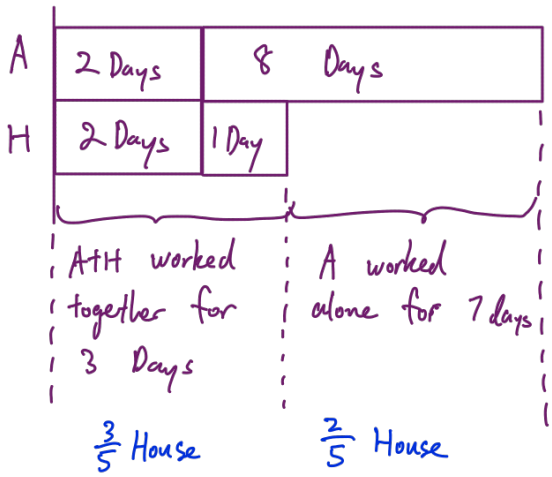


Ans : Sly : 20 days
Ali : 30 days

6. Ahmad and Halim together took 5 days to paint their house.
If Ahmad and Halim work together for 2 days, ✓
followed by Ahmad working alone for 8 days, ✓
Halim will take 1 more day to complete the remaining work. ✓
How long will Ahmad take to paint the house all by himself?

$1 \div \frac{2}{5}$
$= 1 \times \frac{5}{2}$

	Time (Days)	House
A+H	5	1
	1	$\frac{1}{5}$
	3	$\frac{3}{5}$
A	$7 \div \frac{2}{5}$ → $17\frac{1}{2}$	$\frac{2}{5} \div \frac{2}{5}$ → 1



Ans : $17\frac{1}{2}$ Days