Higher Order Thinking Skills

Primary 5

Lesson 1:
Unit Transfer Method (I)

LESSON 1: BEFORE & AFTER SCENARIOS

How to Identify?

There are four basic scenarios where the Before and After may be applied.

SINGLE UNCHANGED

	A	B
Before	10	35
Change	-3	
After	7	35

→ At least one item remains unchanged

TOTAL UNCHANGED

	A	B	Total
Before	10	35	45
Change	-3	+3	
After	7	38	45

→ Same no., opposite signs
∴ Total unchanged

Internal transfer

DIFFERENCE UNCHANGED

	A	B	Diff
Before	10	35	25
Change	-3	-3	
After	7	32	25

→ Same no., same signs
∴ Diff unchanged

same change

ALL CHANGING

	A	B	T	D
Before	10	35	45	25
Change	-3	+5		
After	7	40	47	33

→ Different no.
∴ All Change.

GUIDED EXAMPLE 1   Single Unchanged

B   15   4
John and Freddy shared some stamps in the ratio of 15:4.
C   When their friend, Andy gave Freddy another 55 stamps,
the ratio of Freddy's stamp to John's stamps became 3:5.
How many more stamps did John have more than Freddy at first?

	J	F	Diff
B	15u	4u	11u
[C		+55	]
A	5 ×3 → 15u	3 ×3 → 9u	

$9u - 4u = 55$

$5u = 55$

$1u = 55 \div 5$

$\quad = 11$

$11u = 11 \times 11$

$\qquad = 121$

Ans : 121

GUIDED EXAMPLE 2   Single Unchanged

There are red and green marbles in a bag.
C [When 75 green marbles are added,] $\frac{3}{10}$ $\frac{1}{5}$
the percentage of red marbles decreases from 30% to 20%.
How many red marbles are there in the bag?   B   A

	R	G	
B	3u	7u	
C		+75	
A	1 ×3 → 3u	4 ×3 → 12u	

Red unchanged

$12u - 7u = 75$

$5u = 75$

$1u = 75 \div 5$

$\quad = 15$

$3u = 3 \times 15$

$\quad = 45$

Ans : 45

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GUIDED EXAMPLE 3   Total Unchanged

Henry and Alice have some saving.
B (Henry has 45% less than Alice.]
C (If Alice gives $60 to Henry,]
A (Henry will have 40% less than Alice in the end.]
How much did Henry have at first?

	H	A	Total
B	11 ×8 / 88u	20 ×8 / 160u	31 ×8 / 248u
C	+60	−60	
A	3 ×31 / 93u	5 ×31 / 155u	8 ×31 / 248u

Total unchanged

$93u - 88u = 60$
$5u = 60$
$1u = 60 ÷ 5 = 12$
$88u = 88 × 12 = 1056$

Ans : 1056

GUIDED EXAMPLE 4   Total Unchanged

Denny and Eddie have some stamps in their collection.
B Denny has 8% more stamps than Eddie.
C Eddie decided to give some stamps to Denny.
A As a result, Eddie now has $\frac{1}{7}$ stamps fewer than Denny.
A Given that Eddie has 480 stamps in the end,
find out how many stamps did Eddie give to Denny.

(Direct)

	D	E	Total
B	27 ×20 / 540	25 ×20 / 500	52 ×20 / 1040
C	+?	−?	
A	7 ×80 / 560	6 ×80 / 480	13 ×80 / 1040

Total Unchanged

$500 - 480 = 20$

Ans : 20

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GUIDED EXAMPLE 5   **Difference Unchanged**

There were a group of children in the park.

C   One hour later, 30 boys and 30 girls left the park. $\frac{2}{5}$ $\frac{3}{10}$

As a result, the percentage of boys decreased from 40% to 30%.

How many children were there in the park at first?   B   A

	B	G	Diff	Total
B	2 ×4 8u	3 ×4 12u	1 ×4 4u	20u
C	−30	−30		
A	3u	7u	4u	

Diff   Unchanged

$8u - 3u = 30$

$5u = 30$

$1u = 30 \div 5$

$= 6$

$20u = 20 \times 6$

$= 120$

Ans : <u>120</u>

GUIDED EXAMPLE 6   **Difference Unchanged**

William is 30 years older than his niece, Susan.

How old will William be when he is 4 times as old as Susan?

         4   1

         W   :   S   :   Diff

         4   :   1   :   3   ×10

  ×10   :   30

         40

Ans : <u>40 years old</u>

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GUIDED EXAMPLE 7   All Changing

Kristin has a box of black pens and a box of red pens.

B (The number of black pens is twice the number of red pens.]

C (Kristin removes 4 black pens and 3 red pens from the boxes each time.]

A (After a few rounds, there are 18 black pens and 1 red pen left in the two boxes.]

What was the total number of pens Kristin has?

(NCPS SA1 Q42)

	B	R	Total
B	$2u$	$1u$	$3u$
C	$-4p$	$-3p$	
A	18	1	

$$2u - 4p = 18 \quad (\times 3)$$
$$1u - 3p = 1 \quad (\times 4)$$
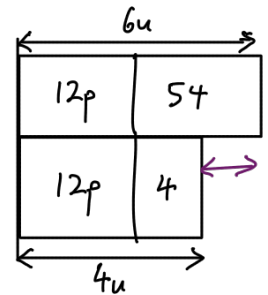
To find u, make p the same.

✓ $[6u - 12p = 54]$

$[4u - 12p = 4]$

$$6u - 4u = 54 - 4$$
$$2u = 50$$
$$1u = 50 \div 2$$
$$= 25$$
$$3u = 3 \times 25$$
$$= 75$$

Ans: $\underline{75}$

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GUIDED EXAMPLE 8 _____ All Changing

B ⎡ There were 4 times as many red pens as blue pens in a box. ⎤
C ⎛ 415 red pens and 46 blue pens were removed from the box. ⎞
A ⎛ As a result, the number of blue pens became thrice the number of red pens. ⎞
⎝ How many red pens were there at the end?

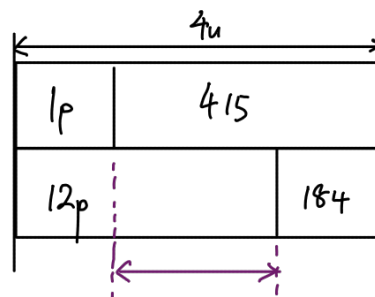|	R	B
B	$4u$	$1u$
C	-415	-46
A	$1p$	$3p$

All Changes

$$\checkmark\ [4u - 415 = 1p]$$

$$1u - 46 = 3p \quad (\times 4)$$

To find p, make u the same.

$$[4u - 184 = 12p]$$

$$12p - 1p = 415 - 184$$

$$11p = 231$$

$$1p = 231 \div 11$$

$$= 21$$

Ans = **21**

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BUILD YOUR UNDERSTANDING

1. B [Mr. Lim is 38 years old and his daughter is 11 years old.]
   C (In how many years' time will Mr. Lim's age be 2.5 times the age of his daughter?)

$\frac{5}{2}$

	Mr Lim	Daughter	Diff
Present	38	11	27
Change	+?	+?	
Future	5 ×9 ↘ 45	2 ×9 ↘ 18	3 ×9 ↘ 27

45 - 38 = 7

Ans : __7 years__

✳ Age Diff Unchanged

2. B (Kenny's saving is $\frac{7}{13}$ of his younger brother, Johnny's saving.)
   C (Kenny received $55 from their mother.)
   A [As a result, the new ratio of Kenny's to Johnny's saving became 3:4.]
   Find out how much more money did Johnny have than Kenny in the end.

	K	J	Diff
B	7 ×4 ↘ 28u	13 ×4 ↘ 52u	
C	+55		
A	3 ×13 ↘ 39u	4 ×13 ↘ 52u	13u

39u - 28u = 55
  11u = 55
  1u = 55 ÷ 11
     = 5
  13u = 13 × 5
     = 65

Ans : __$65__

Johnny Unchanged

3.   Henry and Andy collected some coins.
B   Andy collected ~~25~~% more coins than Henry.
C   If Henry gives 26 coins to Andy,
A   he will have ~~50~~% less than Andy in the end.
        Find out how many coins did Andy have at first.

	H	A	Total
B	4u	5u	9u
C	−26	+26	
A	1 ×3 → 3u	2 ×3 → 6u	3 ×3 → 9u

Total unchanged

$4u - 3u = 26$

$1u = 26$

$5u = 5 \times 26$

$= 130$

Ans : $\underline{130}$

4. B At first, the ratio of Sally's savings to Melvin's savings was 7 : 6.
    C After Sally spent $52 on a bag,
    A the ratio of Sally's savings to Melvin's savings became 5 : 8.
       What was Melvin's savings at first?

	S	M
B	7 ×4 → 28u	6 ×4 → 24u
C	−52	
A	5 ×3 → 15u	8 ×3 → 24u

Melvin Unchanged

$28u - 15u = 52$

$13u = 52$

$1u = 52 \div 13$

$= 4$

$24u = 24 \times 4$

$= 96$

Ans : $\underline{96}$

5. B Jane and Shirley each had an <u>equal amount of money at first.</u>
   C After Jane gave $250 to Shirley,
   A the ratio of Jane's money to Shirley's money was 3 : 8.
   How much money did Jane have in the beginning?

	J	S	Total
B	1 ×11 11u	1 ×11 11u	2 ×11 22u
C	− 250	+ 250	
A	3 ×2 6u	8 ×2 16u	11 ×2 22u

Total Unchanged

$11u - 6u = 250$
$5u = 250$
$1u = 250 \div 5$
$\quad = 50$
$11u = 11 \times 50$
$\quad = 550$

Ans : $550

6. Efron is 30 years younger than Danny.
   B (The ratio of Danny's age to Efron's age now is 8 : 3.)
   C (In how many years' time) will the ratio of Danny's age to Efron's age be 5 : 3?) A

	D	E	Diff
B	8 ×2 16u	3 ×2 6u	5 ×2 10u
C	+ ?	+ ?	
A	5 ×5 25u	3 ×5 15u	2 ×5 10u

Age Diff Unchanged

$10u = 30$
$1u = 30 \div 10$
$\quad = 3$
$25u - 16u = 9u$
$\quad = 9 \times 3$
$\quad = 27$

Ans : 27 years

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7.   There were some children in the swimming pool.
  B The ratio of the number of boys to the number of girls was 3 : 2.
  C When 20 boys had left, the ratio of the number of boys to the
  A total number of children at first became 7 : 15.
     Find the number of boys at the end.

	B	G	Total
B	3 ×3 9u	2 ×3 6u	5 × 3 15u
C	− 20		
A	7u		

$9u - 7u = 20$

$2u = 20$

$1u = 20 \div 2$

$\quad = 10$

$7u = 7 \times 10$

$\quad = 70$

Ans : $\underline{70}$

Girls Unchanged

8. B Benson had 3 times as many beads as Kingsley.
    C After Benson gave 45 beads away and Kingsley lost 7 beads,
    A Kingsley had 3 times as many beads as Benson.
       How many beads did Kingsley have in the end?

	B	K
B	3u	1u
C	~ 45	~ 7
A	1p	3p

$\checkmark \left[3u - 45 = 1p \right]$

$1u - 7 = 3p \quad (×3)$

To find p, make u the same.

$\checkmark \left[3u - 21 = 9p \right]$

$9p - 1p = 45 - 21$

$8p = 24$

$1p = 24 \div 8$

$\quad = 3$

$3p = 3 \times 3$

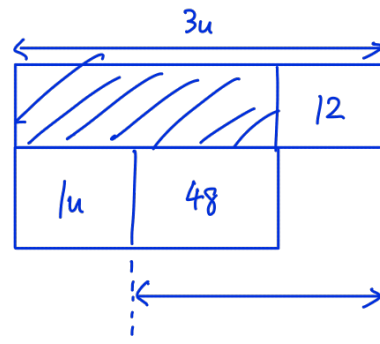$\quad = 9$

Ans : $\underline{9}$

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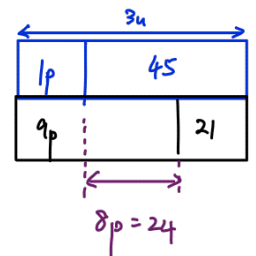
9. B A gardener planted 3 times as many tulips as carnations in the garden.
    C When 12 tulips withered and 48 more carnations were planted in the garden,
    A there was an equal number of each kind of flower.
       What was the total number of tulips at first?

   (Fengshan Pri/P6 Prelim/Q36)

	T	C
B	3u	1u
C	−12	+48
A	1p	1p

All Change

$3u - 12 = 1u + 48$

$$3u - 1u = 48 + 12$$
$$2u = 60$$
$$1u = 60 \div 2$$
$$= 30$$
$$3u = 3 \times 30$$
$$= 90$$

Ans : <u>90</u>

10. **B** There were 55 oranges and 97 mangoes in the carton.
    Ali put more oranges and mangoes were put in the carton.
    C The oranges put in the carton were $\frac{5}{3}$ times as many as the mangoes.
    A As a result, the number of oranges became $\frac{11}{13}$ as many as the number of mangoes.
    Find the <u>number of oranges Ali put in the carton.</u>

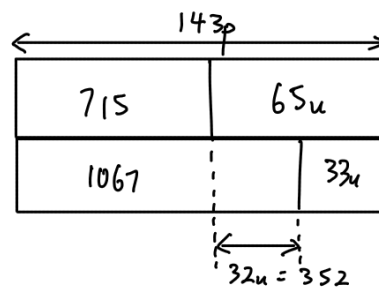|	Or	M
B	55	97
C	+5u	+3u
A	11p	13p

$55 + 5u = 11p \quad (\times 13)$

$97 + 3u = 13p \quad (\times 11)$

To find u, make p the same

$[715 + 65u = 143p]$

$[1067 + 33u = 143p]$

$\overset{143p}{\longleftrightarrow}$

715	65u
1067	33u

$32u = 352$

$65u - 33u = 1067 - 715$

$32u = 352$

$1u = 352 \div 32$

$= 11$

$5u = 5 \times 11$

$= 55$

Ans : <u>55</u>

11. B Shop A has 156 kg of rice. Shop B has 72 kg of rice.
     C After both shops sold an equal amount of rice,
     A the ratio of rice that shop A has to shop B is 4:1.
         Find the amount of rice sold by each shop.

	A	B	Diff
B	156	72	84
C	− ?	− ?	
A	4 ×28 112	1 ×28 28	3 × 28 84

Diff Unchanged

$156 - 112 = 44$

Ans : $\underline{44\ kg}$