

Higher Order Thinking Skills

Primary 5

Lesson 6:
Volume (I)

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LESSON 6 Volume of Cubes and Cuboids

Formula:

(Solid)

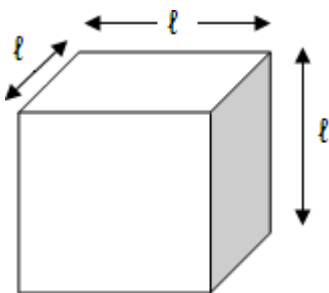
Volume of cuboid = $l \times b \times h$

$= \text{Base area} \times \text{height}$



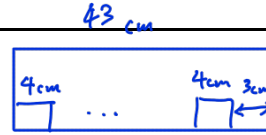
Volume of cube = l^3

$$64 \text{ cm}^3 = 4 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm}$$



GUIDED EXAMPLE 1

What is the maximum number of cubes of edge 4 cm that can be used to fill a tank 43 cm long, 51 cm wide and 39 cm high?



Common mistake:

$$43 \times 51 \times 39 = 85527$$

$$4 \times 4 \times 4 = 64$$

$$85527 \div 64 \approx 1336$$

no definite shape \nearrow Ans: 1336

Consider maximum no. cubes per edge.

$$43 \div 4 = 10 \text{ R}3$$

$$51 \div 4 = 12 \text{ R}3$$

$$39 \div 4 = 9 \text{ R}3$$

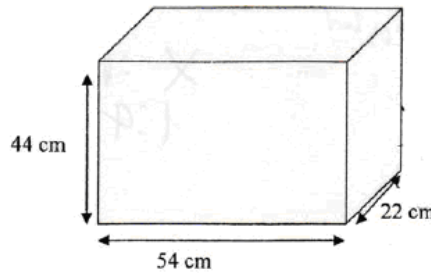
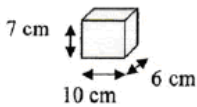
$$10 \times 12 \times 9 = 1080$$

Ans: 1080

GUIDED EXAMPLE 2

Identical boxes measuring 10 cm by 6 cm by 7 cm are put into a rectangular container measuring 54 cm by 22 cm by 44 cm. What is the maximum number of boxes that can be put into the container?

(Catholic High School/SA1/ Q29)

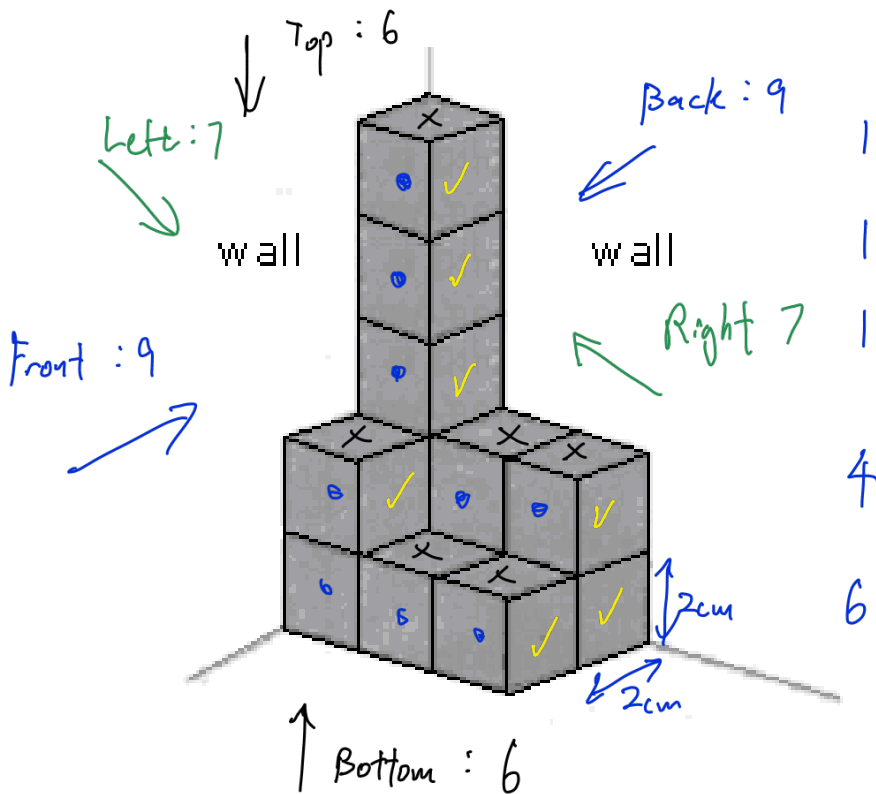


| Orientation | 44 cm | 54 cm | 22 cm | Total no. small cuboids |
|-------------|-------|-------|-------|-----------------------------|
| | 6 | 5 | 3 | $6 \times 5 \times 3 = 90$ |
| | 6 | 9 | 2 | $6 \times 9 \times 2 = 108$ |
| | 4 | 7 | 3 | $4 \times 7 \times 3 = 84$ |
| | 4 | 9 | 3 | $4 \times 9 \times 3 = 108$ |
| | 7 | 5 | 3 | $7 \times 5 \times 3 = 105$ |
| | 7 | 7 | 2 | $7 \times 7 \times 2 = 98$ |
| 3 | | | | Ans: <u>108</u> |

GUIDED EXAMPLE 3

Solid made of cubes

The figure shows a solid leaning against two walls.
The solid is made up of identical cubes of side 2 cm.



(a) Find the volume of the solid.

$$1 + 1 + 1 + 4 + 6 = 13$$

$$2 \times 2 \times 2 = 8$$

$$13 \times 8 = 104$$

Ans : 104 cm³

(b) A layer of paint is being applied on the entire solid.
What is the total area that is being painted?

consider no. faces painted per side.

$$(9 + 7 + 6) \times 2 = 44$$

$$2 \times 2 = 4$$

$$44 \times 4 = 176$$

Ans : 176 cm²

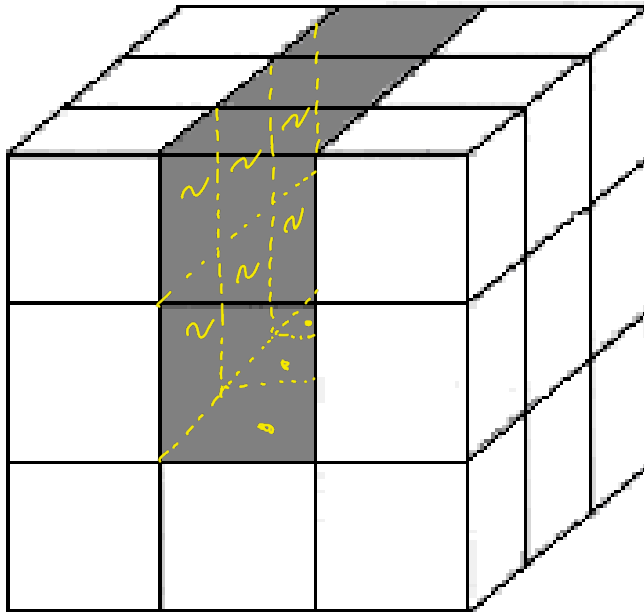
GUIDED EXAMPLE 4

Solid made of cubes

large cube

The figure shows a ~~cube~~ solid made up of some identical unit cubes.

If the six shaded cubes are removed and the remaining solid is painted blue, what is the total area that is being painted?



$$6 \times 9 = 54$$

After removing the 6 cubes,
7 painted faces will be removed.
15 new faces will be exposed.

$$54 - 7 + 3 + 6 + 6 = 62$$

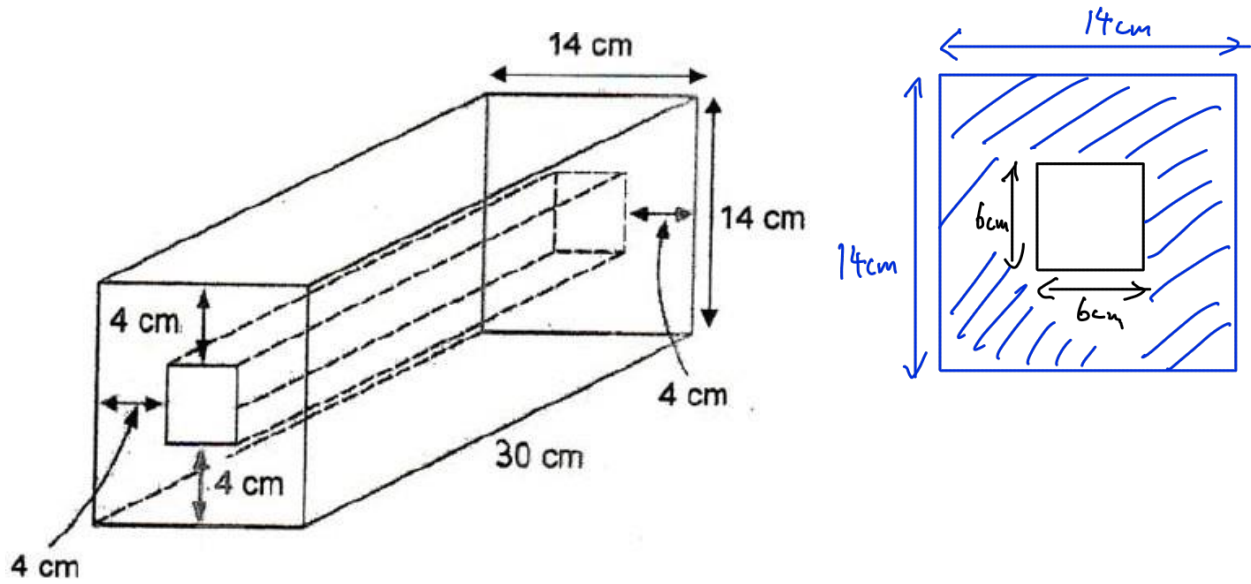
Ans : 62 units²

GUIDED EXAMPLE 5

Cubes and Cuboids

The solid below has a rectangular hole.
Find the volume of the solid.

(Paya Lebar Methodist Girls' / Q40)



$$\begin{aligned} \text{Base area} &= 14 \times 14 - 6 \times 6 \\ &= 160 \end{aligned}$$

$$\begin{aligned} \text{Volume} &= \text{Base Area} \times \text{Height} \\ &= 160 \times 30 \\ &= 4800 \end{aligned}$$

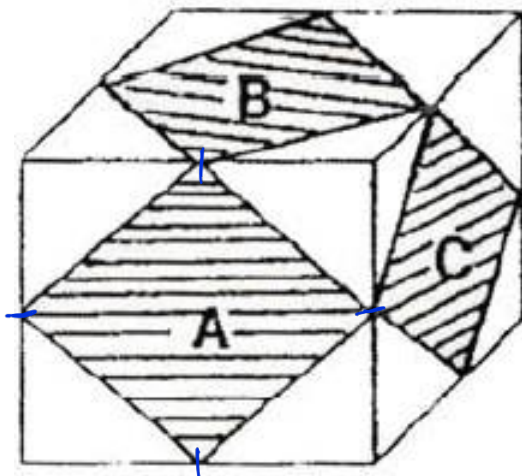
$$\text{Ans} = \underline{4800 \text{ cm}^3}$$

GUIDED EXAMPLE 6

Cubes and Cuboids

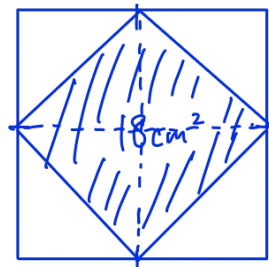
The figure shows a cube with 3 painted parts A, B and C.
 These painted parts are of the same area
 and they are touching the midpoints of the sides of the cube.
The total area of the painted parts is 54cm^2 . Find the volume of the cube.

(Rosyth School/ Prelim/ Q45)



$$54 \div 3 = 18$$

consider 1 of the faces.



$$\begin{aligned} \text{1 whole square} &= 18 \div \frac{1}{4} \\ \text{face area} &= 36 \end{aligned}$$

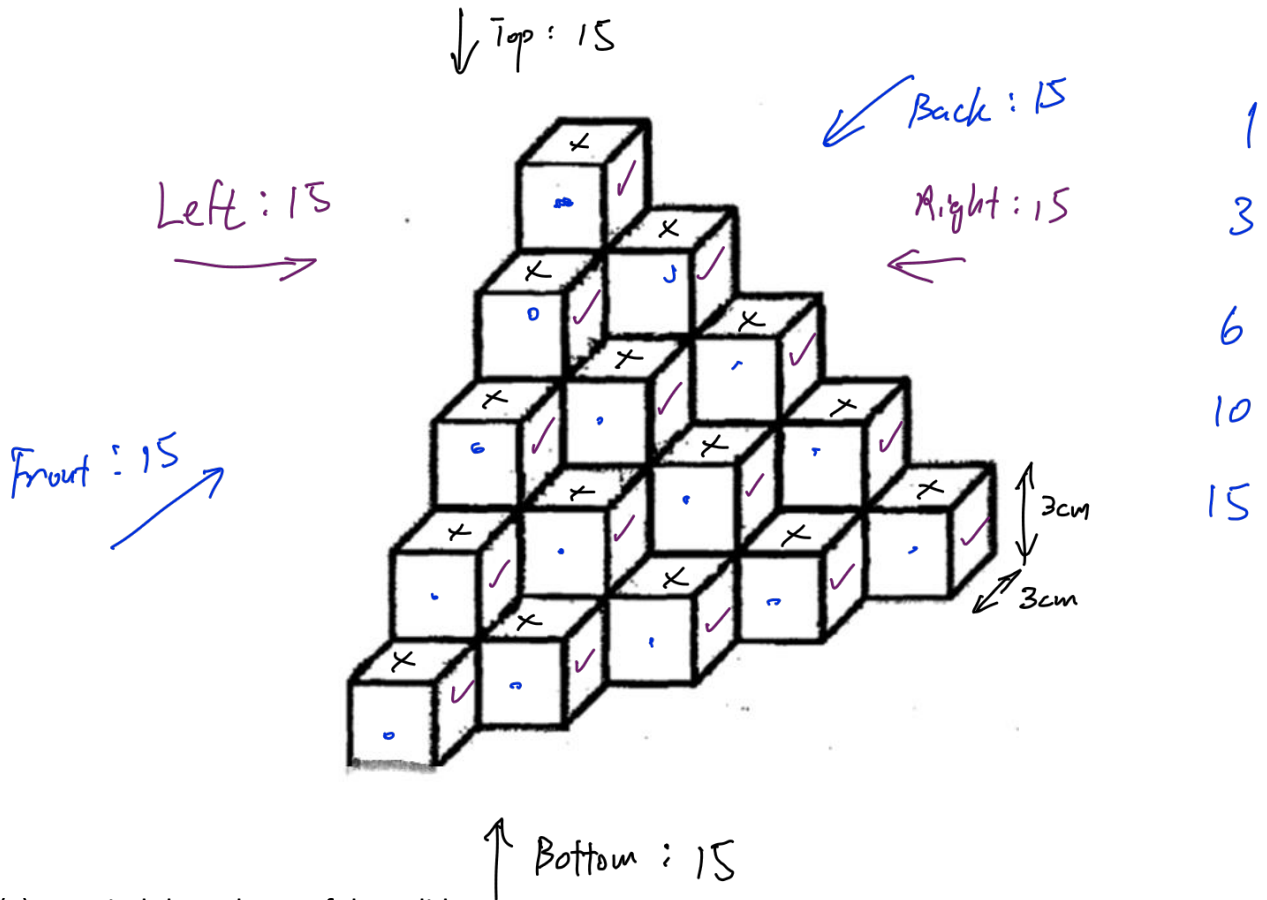
$$\begin{aligned} \text{length of} &= \sqrt{36} \\ \text{square and cube} &= 6 \end{aligned}$$

$$\begin{aligned} \text{Vol of cube} &= 6 \times 6 \times 6 \\ &= 216 \end{aligned}$$

$$\text{Ans : } \underline{216 \text{ cm}^3}$$

BUILD YOUR UNDERSTANDING

1. The figure below is made up of identical 3 cm cubes.



(a) Find the volume of the solid.

$$\begin{aligned}
 1 + 3 + 6 + 10 + 15 &= 35 \\
 3 \times 3 \times 3 &= 27 \\
 35 \times 27 &= 945
 \end{aligned}$$

Ans : 945 cm³

(b) A layer of paint is being applied on the entire solid. What is the total area that is being painted?

consider no. faces painted per side

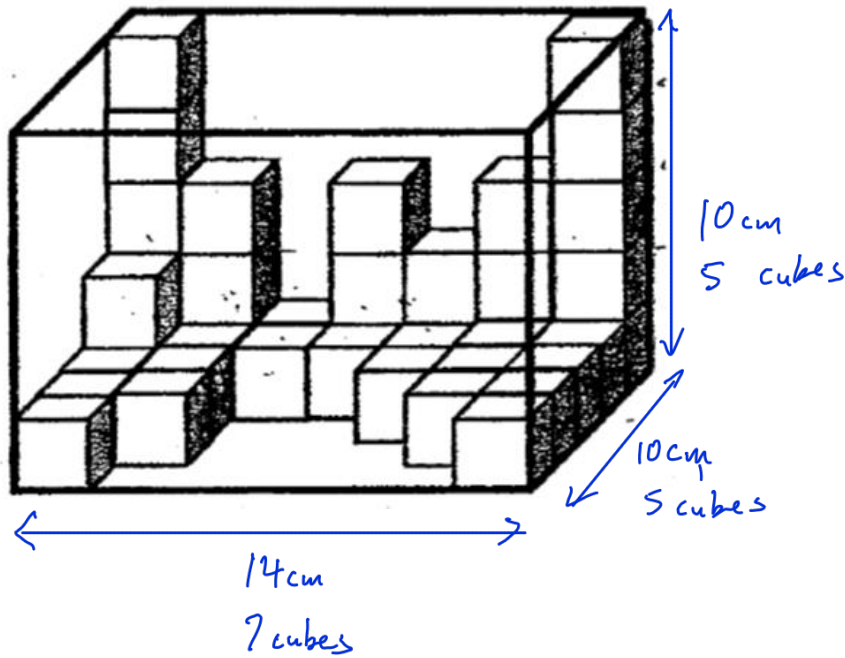
$$6 \times 15 = 90$$

$$3 \times 3 = 9$$

$$90 \times 9 = 810$$

Ans : 810 cm²

2. Derrick put some 2-cm cubes into a rectangular box as shown.



$$\begin{aligned}
 &2 \\
 &2 \\
 &5 \\
 &7 \\
 &5 + 4 + 2 + 2 + 3 + 4 + 5 = 25
 \end{aligned}$$

a) What is the volume of the rectangular box?

$$14 \times 10 \times 10 = 1400$$

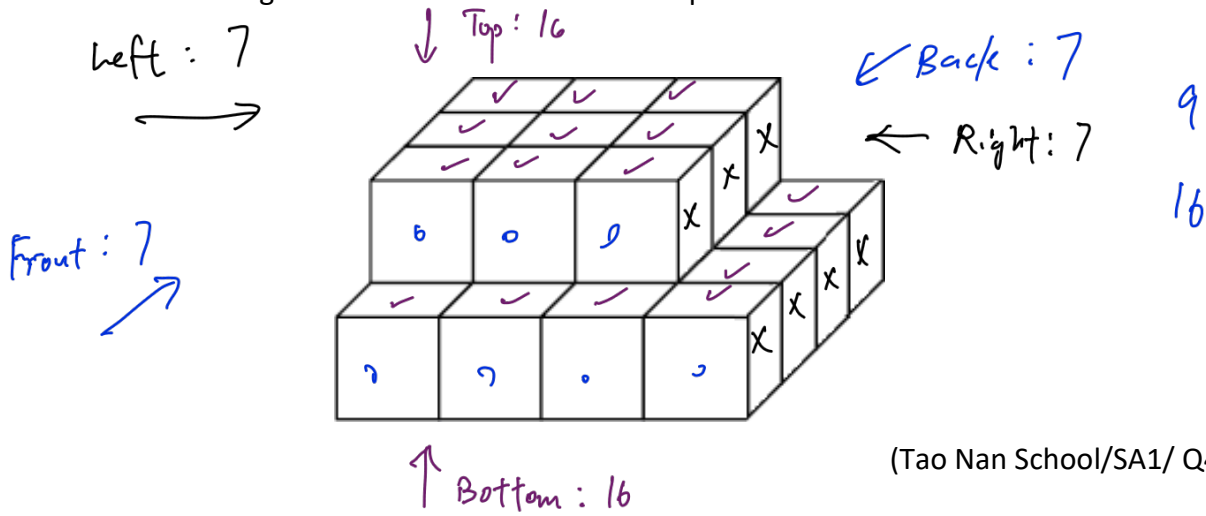
Ans : 1400 cm³

b) How many more of such cubes are needed to fill the box completely?

$$7 \times 5 \times 5 - (2 + 2 + 5 + 7 + 25) = 134$$

Ans : 134

3. The figure below shows a solid made up of 2-cm cubes.



(Tao Nan School/SA1/ Q48)

(a) Find the volume of the solid

$$9 + 16 = 25$$

$$2 \times 2 \times 2 = 8$$

$$25 \times 8 = 200$$

Ans : 200 cm³

(b) Find the total surface area of the solid.

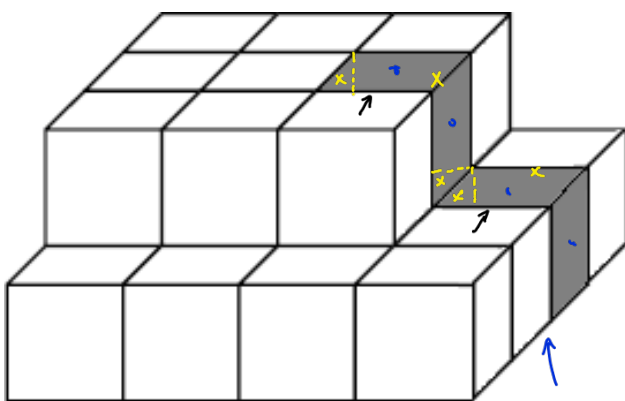
$$2 \times (7 + 7 + 16) = 60$$

$$60 \times 4 = 240$$

$$2 \times 2 = 4$$

Ans : 240 cm²

(c) If the two shaded cubes are removed and the remaining solid is painted red, what is the total area being painted?



$$60 - 5 + 7 = 62$$

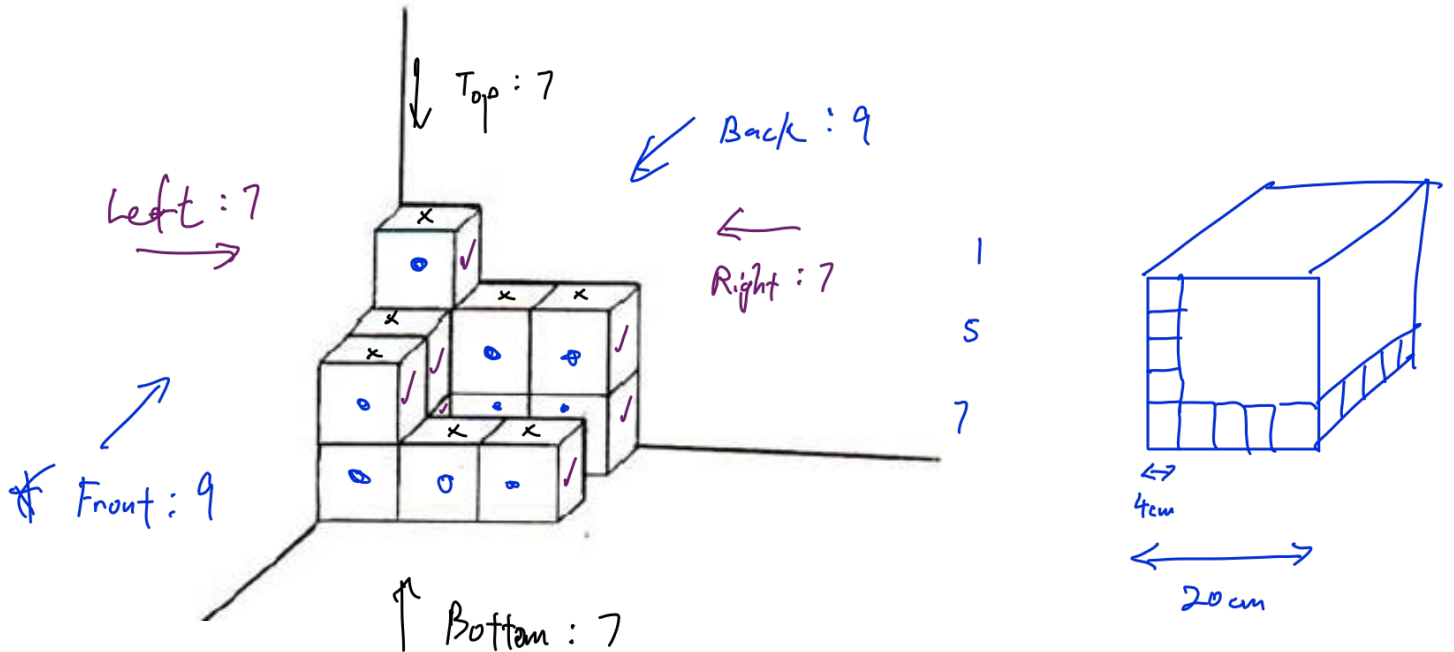
$$62 \times 4 = 248$$

(Remove 5, expose 7 new faces)

Ans : 248 cm²

4. The figure below shows a solid consisting of 4-cm identical cubes.

(St Hilda's Pri/ P6 Prelim/ Q36)



(a) What is the volume of the solid formed?

$$1 + 5 + 7 = 13$$

$$4 \times 4 \times 4 = 64$$

$$13 \times 64 = 832$$

Ans : 832 cm³

(b) How many more cubes must be added to the solid to form a 20-cm cube?

$$20 \div 4 = 5$$

$$5 \times 5 \times 5 = 125$$

$$125 - 13 = 112$$

Ans : 112

(c) If the entire solid is being painted blue, what is the total area that is being painted?

$$2 \times (9 + 7 + 7) = 46$$

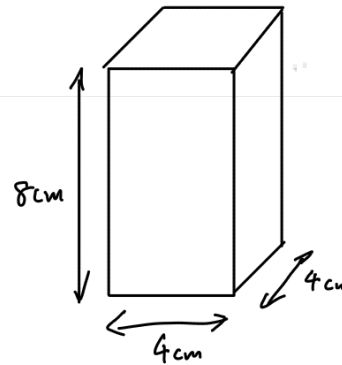
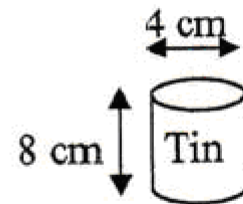
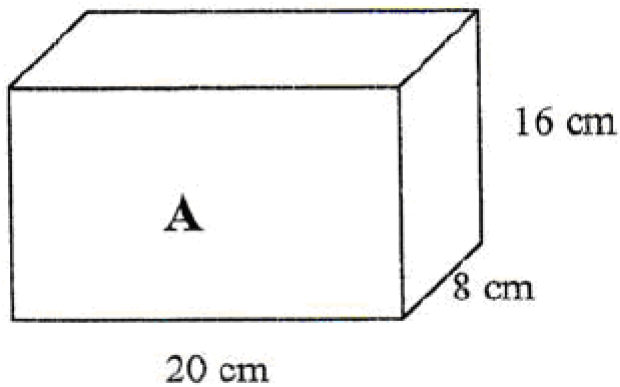
$$4 \times 4 = 16$$

$$46 \times 16 = 736$$

Ans : 736 cm²

5. A is a box measuring 20cm by 8cm by 16cm.
 (Diagram not drawn to scale)
 Bala wants to put as many cylindrical tins as possible into Box A.
 What is the maximum number of cylindrical tins he can put into the box?

(Nan Hua Pri/SA1/ Q27)



consider maximum no. cylinders per edge.

$$16 \div 8 = 2$$

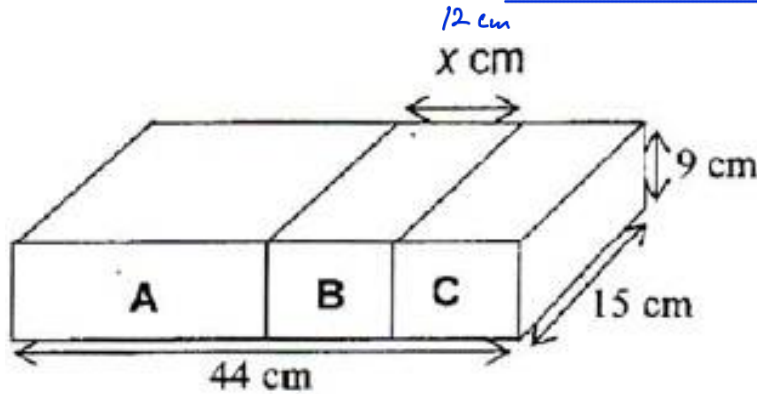
$$20 \div 4 = 5$$

$$8 \div 4 = 2$$

$$2 \times 5 \times 2 = 20$$

Ans : 20

6. A block is 44cm long, 15cm wide and 9cm thick.
 A carpenter cuts the length of the block into three parts A, B and C in the ratio of 5:3:3.



(Casuarina Pri/ SA1/ Q47)

- (a) Find the volume of B

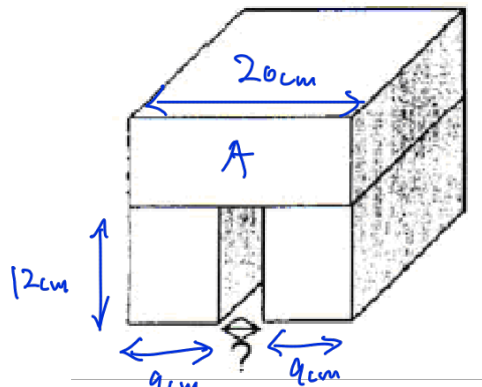
$$\begin{array}{l}
 A : B : C : \text{Total length} \\
 5 : 3 : 3 : 11 \\
 \downarrow \times 4 \\
 20 : 12 : 12 : 44
 \end{array}$$

$$12 \times 15 \times 9 = 1620$$

Ans: 1620 cm³

- (b) Find x

Only 1 possible way to arrange.



Ans: 12

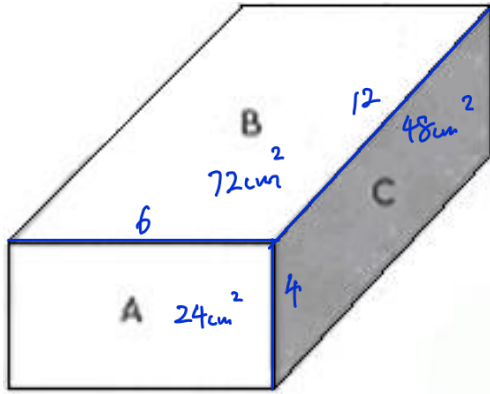
- (c) The carpenter made a stool out of the 3 parts.
 What is the length of the gap between the 2 legs of the stool?

$$20 - 9 - 9 = 2$$

Ans: 2 cm

7. The area of face A, face B and face C of the cuboid shown below is 24 cm^2 , 72 cm^2 and 48 cm^2 respectively.

Given that the sides of the cuboid are whole number,
find the dimension of the cuboid.



Use Factor pair listing to find common edges.

| | | |
|--|--|---|
| 24 $= 1 \times 24$ $= 2 \times 12$ $= 3 \times 8$ $= 4 \times 6$ | 48 $= 1 \times 48$ $= 2 \times 24$ $= 3 \times 16$ $= 4 \times 12$ $= 6 \times 8$ | 72 $= 1 \times 72$ $= 2 \times 36$ $= 3 \times 24$ $= 4 \times 18$ $= 6 \times 12$ $= 8 \times 9$ |
|--|--|---|

Ans : Length : 12cm
Breadth : 6 cm
Height : 4 cm