Higher Order Thinking Skills

Primary 5

Lesson 9:
Pattern

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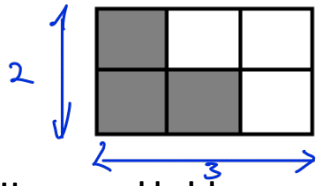
LESSON 9 Pattern

1. Consider the following diagram:



How many black boxes are there in

a) the first row : (1)
the second row : (2)

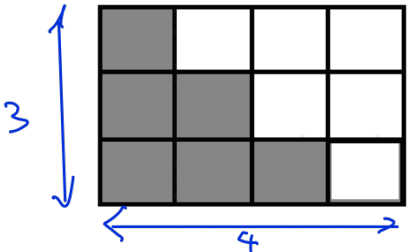
b) Total number of black boxes = (1) + (2)

c) How many black boxes are there altogether?
(Note that exactly half of the squares are black)

Total number of black squares = ½ x (2) x (3)

d) In conclusion: 1 + 2 = ½ x (2) x (3)

2. Consider the following diagram:



How many black boxes are there in

a) the first row : (1)
the second row : (2)
the third row : (3)

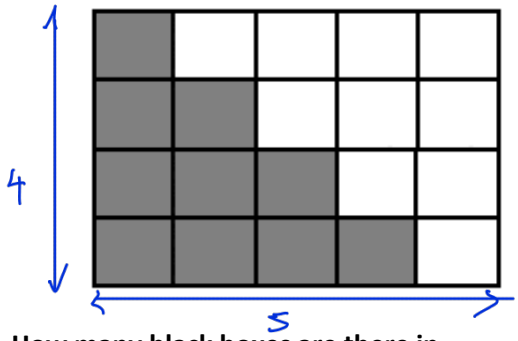
b) Total number of black boxes = (1) + (2) + (3)

c) How many black boxes are there altogether?
(Note that exactly half of the squares are black)

Total number of black squares = ½ x (3) x (4)

d) In conclusion: 1 + 2 + 3 = ½ x (3) x (4)

3. Consider the following diagram:



How many black boxes are there in

a) **the first row : (1)**
the second row : (2)
the third row : (3)
the fourth row : (4)

b) **Total number of black boxes = (1) + (2) + (3) + (4)**

c) **How many black boxes are there altogether?**
(Note that exactly half of the squares are black)

Total number of black squares = ½ x (4) x (5)

d) **In conclusion: 1 + 2 + 3 + 4 = ½ x (4) x (5**

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4. Complete the following:

1 + 2 **= ½ x (2) x (3)**

1 + 2 + 3 **= ½ x (3) x (4)**

1 + 2 + 3 + 4 **= ½ x (4) x (5)**

1 + 2 + 3 + 4 + 5 **= ½ x (5) x (6)**

1 + 2 + 3 + 4 + 5 +.....100 **= ½ x (100) x (101)**

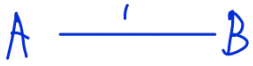
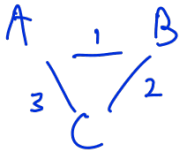
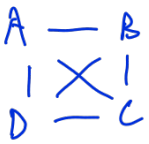
1 + 2 + 3 + 4 + 5 +..... (last Number) **= ½ x (N) x (N+1)**

N

GUIDED EXAMPLE 1

How many different handshakes can be made in a party of;

* Last no. = no. children − 1

		No. HS	Restated form
a)	2 children? 	1	1
b)	3 children? 	3	1 + 2
c)	4 children? 	6	1 + 2 + 3
d)	100 children?	$\frac{1}{2} \times 99 \times 100$ = 4950	1 + 2 + 3 + ... + 98 + 99

e) Given that there a total of 1225 handshakes made in the party, how many children are there in the party?
(Assume that each child shakes hands with every other child only once)

Use GRC or trial & error

$\frac{1}{2} \times 30 \times 31 = 465$ ✗

$\frac{1}{2} \times 40 \times 41 = 820$ ✗

$\frac{1}{2} \times 50 \times 51 = 1275$ ✗

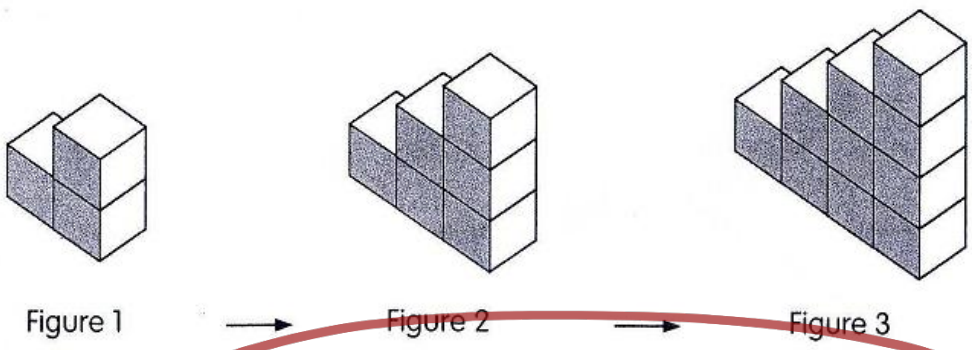
$\frac{1}{2} \times 49 \times 50 = 1225$ ✓

$1225 = 1 + 2 + ... + 48 + 49$

No. children = 49 + 1
= 50

GUIDED EXAMPLE 2

Study the pattern below and complete the table:



* last no. = FN + 1

(a) Complete the following table:

Figure number	Number of cubes
1	(3)
2	(6)
3	(10)
4	(15)
5	(21)

+3, +4, +5, +6

Rewrite
1 + 2
1 + 2 + 3
1 + 2 + 3 + 4
+1

$\frac{1}{2} \times 101 \times 102$

1 + 2 + ... + 101

(b) Calculate the number of cubes in Figure 15.

$1 + 2 + \ldots + 15 + 16 = \frac{1}{2} \times 16 \times 17 = 136$

(c) Calculate the number of cubes in Figure 50.

$1 + 2 + \ldots + 50 + 51 = \frac{1}{2} \times 51 \times 52 = 1326$

(d) Which Figure number has 5151 cubes?

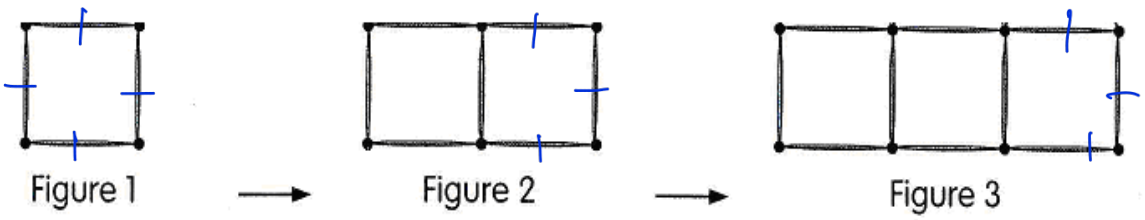
Trial & error

$\frac{1}{2} \times 101 \times 102 = 5151$ ✓

$101 - 1 = 100$

GUIDED EXAMPLE 3

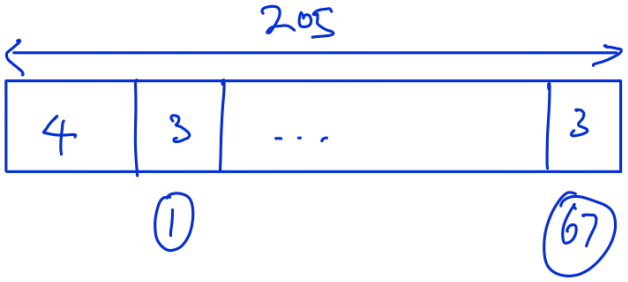
Study the pattern below and fill up the table:



Complete the following table:

* No. +3's = FN − 1

Figure	Number of sticks	
1	(4)	4
2	(7)	4 + 1×3
3	(10)	4 + 2×3
4	(13)	4 + 3×3
5	(16)	
:	:	
50	(151)	4 + 49 × 3
:	:	
(68)	**205**	4 + 67 × 3



205 − 4 = 201

201 ÷ 3 = 67

67 + 1 = 68

GUIDED EXAMPLE 4

Study the pattern below and fill up the table:

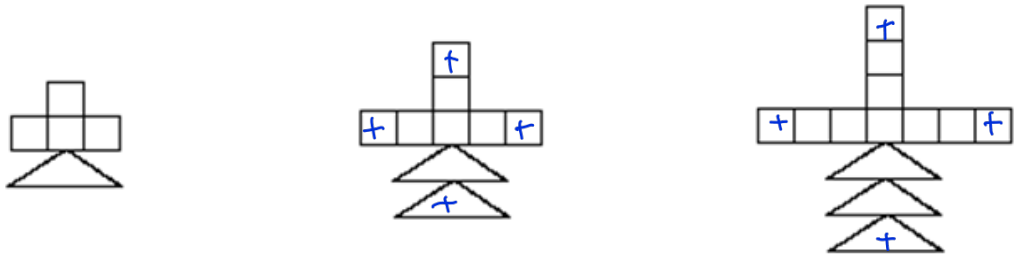


Figure 1 → Figure 2 → Figure 3

Complete the following table:

Figure	Total Numbers of Triangles	Total Numbers of Squares	Total numbers of Triangles and Squares
1	1	(4)	(5)
2	2	(7)	(9)
3	3	(10)	(13)
4	(4)	(13)	(17)
5	(5)	(16)	(21)
.....	.....	.....	.....
(14)	(14)	43	(57)
.....	.....	.....	.....
25	(25)	(76)	(101)
.....	.....	.....	.....
(30)	(30)	(91)	121
.....	.....	.....	.....
40	(40)	(121)	(161)

5

5 + 1 × 4

5 + 2 × 4

+3 +3 +3 +3

+1 +1

4 + 13 × 3 = 43

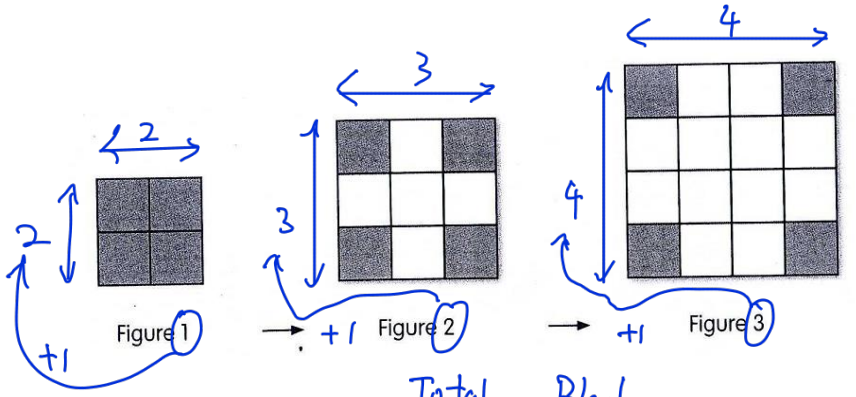
4 + 24 × 3 = 76

5 + 29 × 4

4 + 39 × 3 = 121

GUIDED EXAMPLE 5

Study the pattern below and fill up the table:



Complete the following table:

Total - Black

Figure	Number of black squares	Number of white squares	Total number of squares	
1	(4)	(0)	(4)	2x2
2	(4)	(5)	(9)	3x3
3	(4)	(12)	(16)	4x4
4	(4)	(21)	(25)	5x5
5	(4)	(32)	(36)	6x6
: :	: :	: :	: :	
10	(4)	(117)	(121)	11x11
: :	: :	: :	: :	
? 11	(4)	**140**	(144)	12x12
: :	: :	: :	: :	
(24)	(4)	(621)	**625**	25x25
: :	: :	: :	: :	
30	(4)	(957)	(961)	31x31

BUILD YOUR UNDERSTANDING

Find the sum of the following:

1

a) 1 + 2 + ... + 18 + 19

$\frac{1}{2} \times 19 \times 20^{10} = 190$

b) (1 + 2 + ... + 179 + 180) + (180 + 179 + ... + 2 + 1)

$2 \times \frac{1}{2} \times 180 \times 181 = 32580$

c) 2 + 4 + 6 + ... + 398 + 400

1	2	...	199	200
1	2	...	199	200

} 20 100
} 20 100

2 + 4 ... + 398 + 400

$1 + 2 + 3 + ... + 199 + 200$

$= \frac{1}{2} \times 200 \times 201$

$= 20\,100$

$2 \times 20\,100 = 40\,200$

d) 7 + 14 + 21 + ... + 490 + 497

÷7 (

$1 + 2 + 3 + ... + 70 + 71$

$= \frac{1}{2} \times 71 \times 72$

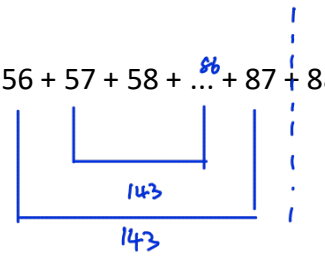
$= 2556$

$7 \times 2556 = 17892$

57 − 56 = 1

56 ⟷ 57
1 gap

e) 56 + 57 + 58 + ... + 87 + 88



No. terms = 88 − 56 + 1 = 33 (odd)

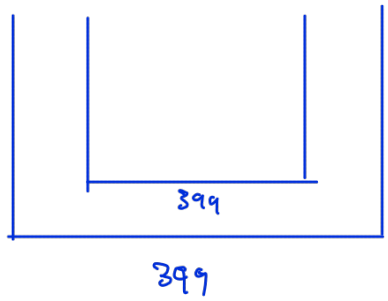
1 group = 56 + 87 = 143

No. groups = (33 − 1) ÷ 2 = 16

Total = 16 × 143 + 88

= 2376

f) 99 + 100 + 101 + ... + 299 + 300



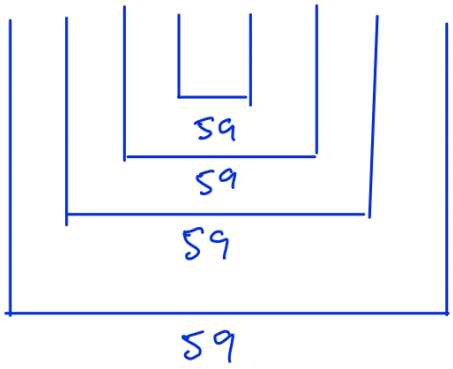
No. terms = 300 − 99 + 1 = 202

1 group = 300 + 99 = 399

No. groups = 202 ÷ 2 = 101

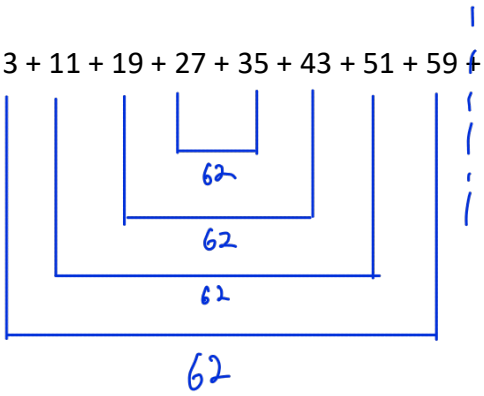
Total = 101 × 399 = 40 299

g) 5 + 12 + 19 + 26 + 33 + 40 + 47 + 54



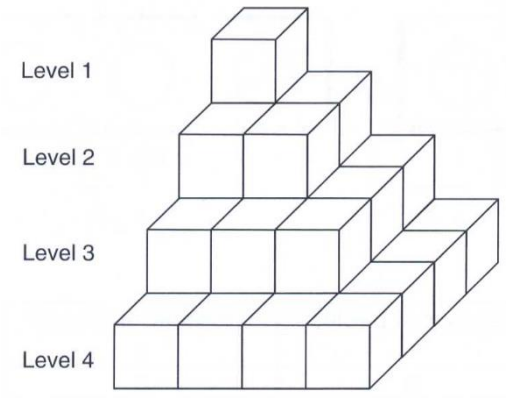
4 × 59 = 236

h) 3 + 11 + 19 + 27 + 35 + 43 + 51 + 59 + 67



4 × 62 + 67 = 315

2. Study the pattern of the structure below.



(a) Fill in the missing number of cubes.

Listing

Level	Number of cubes	Total
1	(1)	(1)
2	(4)	(5)
3	(9)	(14)
4	(16)	(30)
5	(25)	(55)
:	:	:
10	(100)	(385)

(b) If more cubes are to be added to build the structure,
how many cubes are needed to use for Level 10?

$55 + 6\times6 + 7\times7 + 8\times8 + 9\times9 + 100 = 385$

(c) If a total of 506 cubes are used to build the whole structure,
how many levels are there in the structure?

Level	Total no. cubes
10	385
11	$385 + 11\times11 = 506$

3. Study the pattern below.

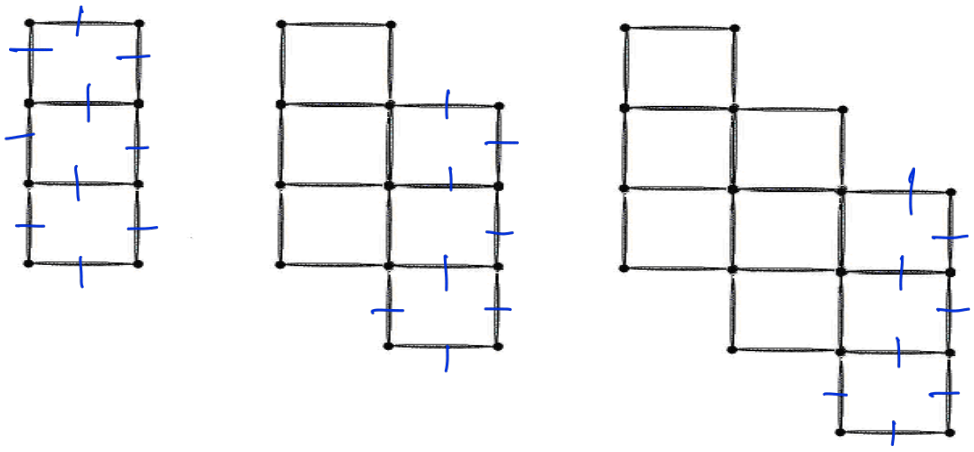


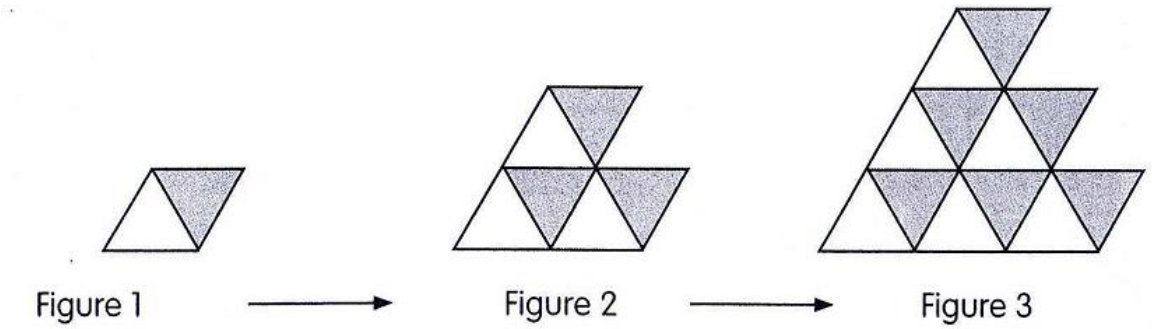
Figure 1 → Figure 2 → Figure 3

Each ☐ represents a square. Complete the following table.

No. +8's = FN − 1

Figure	Number of Squares	Number of sticks	
1	(3)	(10)	10
2	(6)	(18)	10 + 1×8
3	(9)	(26)	10 + 2×8
4	(12)	(34)	10 + 3×8
5	(15)	(42)	
:	:	:	
50	(150)	(402)	10 + 49×8
	:	:	
(100)	300	(802)	10 + 99×8
:	:	:	
(111)	(333)	890	10 + 110×8

4. Study the pattern below.



Complete the following table.

Figure	Number of white triangles	Number of black triangles	Total number of triangles
1	(1)	(1)	(2)
2	(3)	(3)	(6)
3	(6)	(6)	(12)
4	(10)	(10)	(20)
5	(15)	(15)	(30)
:	1+2+... + 29 + 30	:	:
30	(465)	(465)	(930)
:	1+2+... + 80	:	:
(80)	(3240)	(3240)	6480
:	1+2+... + 120	:	:
(120)	(7260)	(7260)	14520

$\frac{1}{2} \times N \times (N+1)$

5. Study the pattern in the table and answer the questions below:

perfect squares

Pattern	Statement	Sum	
1	1	1	1×1
2	1 + 2 + 1	4	2×2
3	1 + 2 + 3 + 2 + 1	9	3×3
4	1 + 2 + 3 + 4 + 3 + 2 + 1	16	4×4
? 30		900	30×30

a) Write down the statement for pattern 6.

1+2+3+4+5+6+5+4+3+2+1

b) Calculate this sum.

36

c) When the sum is 900,
what is the pattern number?

$\sqrt{900} = 30$

6. Study the pattern below and complete the following table:

Sequence	Number
1	4
2	7
3	10
4	(13)
:	:
:	:
55	(166)
:	:
(68)	205

+3 +3 +3

4

4 + 1x3

4 + 2x3

4 + 54 x 3

4 + 67 x3