**Secondary 2 Mathematics | Lesson 7**

**Expansion and Factorisation of Algebraic Expressions**

**Learning objectives:**

- Factorise quadratic expressions
- Factorise algebraic expressions of the form $ab + ad + bc + bd$

**Prerequisites:**

Expand and factorise linear expressions

Factorisation is the reverse of expansion.

**Guided Example 1 – Factorising linear expressions (Recap)**

Factorise each of the following expressions completely.

a) $12x + 8$ b) $-15x - 25$ c) $-27ax + 12ay$ d) $36p - 54pq + 18pr$

**Exercise 1**

Factorise each of the following expressions completely.

a) $21 + 35a$ b) $-8 - 20p$ c) $-42xy - 12xz$ d) $-9z - 24bz - 15cz$

**Guided Example 2 – Factorising expressions involving squares and cubes**

Factorise each of the following expressions completely.

a) $10x^2 + 8x$ b) $-49b - 28b^2$ c) $x^2yz^3 - yz^2$

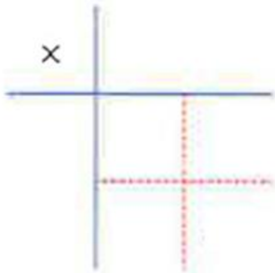
**Exercise 2**

Factorise each of the following expressions completely.

a) $10a^2 - 15a$ b) $2\pi r^2 + 2\pi rh$ c) $c^2d^3 + c^3d^2 - c^2d^2$

**How to factorise quadratic expressions using the multiplication frame**

Let us start with expansion of a relatively simple expression,



$(x+2)(x+3)$

$= x^2 + 3x + 2x + 6$

$= x^2 + 5x + 6$

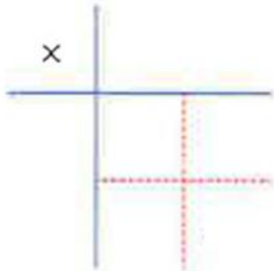
How do we factorise $x^2 + 5x + 6$?

Breaking down $x^2 + 5x + 6$ into $x^2 + 3x + 2x + 6$ allows us to factorise by grouping more easily. However, there are certain constraints in how the $bx$ term must be broken down into its sum pairs, and the sum pairs are often not immediately obvious.

We can see that the $ax^2$ and $c$ terms remain unchanged.

*There is some trial and error involved. Be calm in execution because it can get frustrating.*

1) Write the $x^2$ and $+6$ terms in their respective boxes.
   How did the $x^2$ and $+6$ come about?
   They were obtained after being multiplied by a particular factor pair.
   We will need to use consider their possible factor pairs using listing.

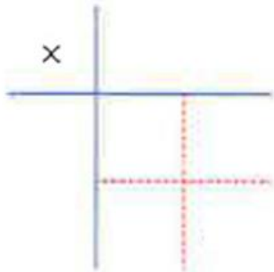


   $x^2 = x \times x$

   $6 = 1 \times 6 = (-1) \times (-6)$
   $= 2 \times 3 = (-2) \times (-3)$

   When $a$ in the $ax^2$ term is 1 or prime, or when $c$ is 1 or prime, listing factor pairs becomes convenient.

2) Fix either the $ax^2$ factors or the $c$ factors first, then use trial and error.
   Since $x^2 = x \times x$, there is only one unique combination and they can be listed out first. Now, we need to put in numbers into these 2 boxes and they must fulfill these conditions:
   i) They must be numbers of a factor pair of 6
   ii) The product of the numbers and the $x$ term must sum up to $bx$. In this case, $+5x$.



3) Filter out possibilities with reasoning.

Based on the factor pairs of 6, there are 4 unique combinations.

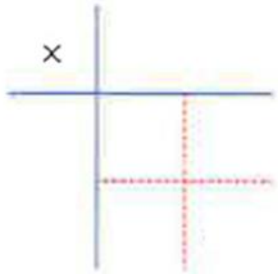
To get a sum pair of $+5x$, we can reason that both factors must be positive.

That leaves us with $1 \times 6$ and $2 \times 3$ to consider.

If we were to test out $(x + 1)(x + 6)$, this yields

$x^2 + 6x + x + 6 = x^2 + 7x + 6$,

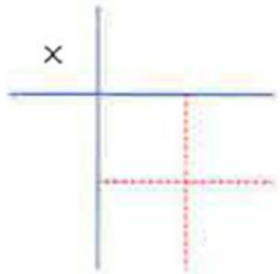
but $+7x \neq +5x$. $\therefore$ this combination does not work.




If we were to test out $(x + 2)(x + 3)$, this yields

$x^2 + 3x + 2x + 6 = x^2 + 5x + 6$,

and the $bx$ term is $+5x$. $\therefore$ this combination works.




To summarise,

1) Break down the $ax^2$ and $c$ term into their factor pairs.
2) Find a combination such that the product of the terms involving $x$ add up to $bx$.

**Guided Example 3 – Factorising expressions using Multiplication Frame**

Factorise each of the following quadratic expressions.

a) $x^2 + 6x + 5$

b) $x^2 - x - 12$

**Exercise 3**

Factorise each of the following quadratic expressions.

a) $x^2 + 2x - 15$

b) $x^2 + 5x - 14$

**Guided Example 4 – Factorising expressions using Multiplication Frame**

Factorise each of the following quadratic expressions.

a) $2x^2 + 7x + 6$

b) $6y^2 - 11y + 4$

**Exercise 4**

Factorise each of the following quadratic expressions.

a) $3x^2 + 10x - 8$

b) $7 - 13x - 2x^2$

Two expressions are equivalent if and only if the coefficients of each term are the same.

If $a_1x^2 + b_1x + c_1 = a_2x^2 + b_2x + c_2$, then $a_1 = a_2, b_1 = b_2$ and $c_1 = c_2$ and

if $a_1 = a_2, b_1 = b_2$ and $c_1 = c_2$, then $a_1x^2 + b_1x + c_1 = a_2x^2 + b_2x + c_2$.

**Guided Example 5 – Checking whether two expressions are equivalent**

Without expanding $(2x + 3)(x - 5)$ completely, and without factorising $3x^2 - 18x + 15$, explain why the two expressions are not equivalent.

**Exercise 5**

Dina says that $(y - 2)(2y + 1)$ and $4y^2 + 7y - 2$ are equivalent because the constant term of $(y - 2)(2y + 1)$ is also $-2$. Without expanding or factorising the expressions, explain if you agree with this statement.

**Guided Example 6 – Factorising quadratic expressions in two variables using multiplication frame**

Factorise each of the following expressions completely.

a) $x^2 + 2xy - 8y^2$

b) $6x^2 + 11xy + 5y^2$

**Exercise 6**

Factorise each of the following expressions completely.

a) $x^2 - 2xy - 15y^2$

b) $6x^2 - 21xy + 18y^2$

**Guided Example 7 – Factorising algebraic expressions into the form $(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{c}+\boldsymbol{d})$**

Factorise each of the following expressions completely.

a) $ab + ac + 2bd + 2cd$

b) $6ax - 20by - 8bx + 15ay$

**Exercise 7**

Factorise each of the following expressions completely.

a) $3pq + 7rs + 3pr + 7qs$

b) $3hp - 12kq + 18kp - 2hq$

**Guided Example 8 – Factorising algebraic expressions into the form $(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{c}+\boldsymbol{d})$**

Find the two factors in $6xy - 15x + 20 - 8y$.

**Exercise 8**

Find the two factors in $6ab - 9ac + 21c - 14b$.

**Guided Example 9 – Factorising quadratic expressions by grouping**

Factorise $x^2 + xy - 3x - 3y$ completely.

**Exercise 9**

Factorise $15w^2 - 20w - 6wz + 8z$ completely.

**Guided Example 10 - Applications**

By factorising the expression $3x^2 + 26x + 51$, find two factors of 32651.

**Exercise 10**

Factorise $4x^2 + 13x + 3$ and use your result to find the prime factors of 41303.

## Additional Practice for Self-Revision

### Additional Practice 1

Factorise each of the following expressions completely.

a) $a^2 + 20a + 75$

b) $b^2 + 19b + 18$

c) $c^2 - 11x + 28$

d) $d^2 - 21d + 68$

**Additional Practice 2**

Factorise each of the following expressions completely.

a) $6a^2 + 31a + 5$

b) $8b^2 + 30b + 27$

c) $4c^2 - 25c + 6$

d) $9d^2 - 36d + 32$

**Additional Practice 3**

Factorise each of the following expressions completely.

a) $a^2 + 7ab + 6b^2$

b) $c^2 + 11cd - 12d^2$

c) $2d^2 - de - 15e^2$

d) $6f^2 - 29fg + 28g^2$

### Additional Practice 4

Factorise each of the following expressions completely.

a) $p^2 + pq + 3qr + 3pr$

b) $3xy + 6y - 5x - 10$

c) $x^2z - 4y - x^2y + 4z$

d) $x^3 + xy - 3x^2y - 3y^2$

### Answers to Additional Practice

| Question | Final Answer | Question | Final Answer |
|---|---|---|---|
| **1a** | $(a+15)(a+5)$ | **3a** | $(a+b)(a+6b)$ |
| **1b** | $(b+18)(b+1)$ | **3b** | $(c+12d)(c-d)$ |
| **1c** | $(c-7)(c-4)$ | **3c** | $(2d+5e)(d-3e)$ |
| **1d** | $(d-17)(d-4)$ | **3d** | $(3f-4g)(2f-7g)$ |
| **2a** | $(6a+1)(a+5)$ | **4a** | $(p+3r)(p+q)$ |
| **2b** | $(4b+9)(2b+3)$ | **4b** | $(3y-5)(x+2)$ |
| **2c** | $(4c-1)(c-6)$ | **4c** | $(x^2+4)(z-y)$ |
| **2d** | $(3d-8)(3d-4)$ | **4d** | $(x^2+y)(x-3y)$ |